GENERAL MATHEMATICS 11

Name of Learner:________________ Grade Level:______

Section:________________________ Date:____________

LEARNING ACTIVITY SHEET

DETERMINING THE INTERCEPTS, ZEROES AND ASYMPTOTES OF RATIONAL FUNCTIONS

Background Information for Learners

Rational Functions can be written in the form f(x)= N(x)/D(x) where N(x) and D(x) are polynomials and D(x) is not the zero polynomial.

The intercepts are the x-or y- coordinates of the points at which the graph crosses the x-axis of y-axis respectively. The zeros of the rational function f are the values of the independent variable that make the numerator zero but are not restrictions of the rational function f. Moreover, the asymptote is a line (or a curve) that the graph of a function gets close to but not touch either x- or y-axis.

Remember:

1. To find the x-intercept, substitute 0 for y and solve for x.
   To find the y-intercept, substitute 0 for x and solve for y.
2. To find the zeros of rational functions
   - Factor the numerator and the denominator of the rational function f if possible
   - Identify the restrictions of the rational function f. (The restrictions are the values of the independent variable that make the denominator equal to zero.)
   - Identify the values of the independent variable that make the numerator equal to zero.
   - The zeros of the rational function f are the values of the independent variable that make the numerator zero but are not restrictions of the rational function f.
3. On vertical asymptotes they are the restrictions on the x-values of a reduce rational function by equating the denominator to 0 and solve for x.

   On horizontal Asymptotes
   - If $n<m$, the graph of f has the line $y=0$
   - If $n=m$, the graph of f has the line $y=a_n/b_m$ where a_n and b_m are the leading coefficients of the numerator and denominator, respectively
   - If $n>m$, the graph has no horizontal asymptote.

Learning Competency

Determines the (a) intercepts; (b) zeroes (c) asymptotes of rational functions (Quarter 1, Week 2, M11GM-1c-1)

Note: Practice Personal Hygiene protocols at all times

Illustrative Examples:

A. Which of the following are rational functions?

1. f(x)= $(x+2)/(1+x)$
2. f(x)= $(x^2+14x-15)/(x+7)$
3. g(x)=$(x^2+\sqrt{x})/(2x^2-1)$
4. g(x)= $5+x/x^2+\sqrt[3]{x}+1$

Solution:

1. Rational Function: Both numerator and denominator are polynomials
2. Rational Function: Both numerator and denominator are polynomials
3. Not a Rational Function: Numerator is not a polynomial
4. Not a Rational Function: Denominator is not a polynomial

B. Find the **x -and y intercepts, zeros** and the **asymptotes** of

1. f(x)= $(x+6)/(x-3)$

Solution:

For x intercept

f(x)= $(x+6)/(x-3)$

0 = $(x+6)/(x-3)$ Substitute 0 for y or f(x)

0= $(x+6)$ Multiply both sides by x-3

x= -6 Simplify

Therefore, the x-intercept is -6 or (-6,0)

For y intercept

f(x)= $(x+6)/(x-3)$

f(0)= $(0+6)/(0-3)$ Substitute 0 for x

= $6/-3$ Simplify

= -2

Therefore, The y intercept is -2 or (0,-2)

For the zeros

f(x)= $(x+6)/(x-3)$

$x-3=0$

x=3 Identify restrictions by making the denominator equal to 0

$x+6=0$

x=-6 identify the values of x that make the numerator equal to 0

Since -6 make the numerator zero but not restrictions of the rational function f, therefore **-6 is the zero of the function.**

Note: Practice Personal Hygiene protocols at all times

Vertical asymptote

$$x - 3 = 0$$

x=3

Horizontal asymptote

y= 6/-3

y = -2

2. f(x)= $(x^2 - 9)/(x^2 - x - 6)$
   For x intercept

f(x)= $(x^2 - 9)/(x^2 - x - 6)$	Factor
f(x)= $(\mathbf{x} - \mathbf{3})(x + 3)/\ (\boldsymbol{x} - \mathbf{3})/(x + 2)$	Cancel $(\mathbf{x} - \mathbf{3})$
f(x)= $(x + 3)/\ (x + 2)$	
f(0)= $(x + 3)/\ (x + 2)$	Substitute 0 for y or f(x)
0=$(x + 3)$	Multiply both sides by $(x + 2)$
x=-3	Simplify

Therefore, the x intercept is -3

For y intercept

f(x)= $(x^2 - 9)/(x^2 - x - 6)$

f(0)= $(0^2 - 9)/(0^2 - 0 - 6)$	Substitute 0 for x
= -9/-6	Simplify
=3/2	

Therefore, the y intercept is 3/2

For the zeros

f(x)= $(x^2 - 9)/(x^2 - x - 6)$

f(x)= $(\mathrm{x} - 3)(x + 3)/\ (x - 3)/(x + 2\ )$	Factor the numerator and denominator of f
f(x)= $(\mathbf{x} - \mathbf{3})(x + 3)/\ (\boldsymbol{x} - \mathbf{3})/(x + 2)$	Cancel $(\mathbf{x} - \mathbf{3})$
$x = 3\ \ and\ \ x = -2$	Identify restrictions by making the denominator equal to 0
$x = 3\ \ and\ \ x = -3$	identify the values of x that make the numerator equal to 0
	Since -3 make the numerator zero but not restrictions of the rational function f, **therefore -3 is the zero of the function**

Note: Practice Personal Hygiene protocols at all times

Vertical Asymptote

$x = 3 \ and \ x = -2$

Horizontal asymptote: Y=1

Exercise 1: Know Me Better

Directions: Identify which of the following are rational functions.

5. f(x)= $(4 + x)/(x + 3)$
6. f(x)= $(x^2 - 4x - 5)/(x - 1)$
7. g(x)=$(\sqrt{x} + 3)/(x^2 - 1)$
8. i(x)= $5 + x - 4x^2/x^2 + \sqrt[3]{2x} + 1$

Exercise 2. Place Me on the Table

Directions: Find (a) the **zeros**; (b) the **x-and y-intercepts** and (c) the **asymptotes** of the following rational functions.

Rational Function	Zeros	x-and y-Intercepts	Asymptotes
1. f(x)= $(x - 5)/(x + 2)$			
2. f(x)= $(x - 5)/(x^2 - 25)$			
3.f(x)= $(x^2 - 5x + 4)/(x^2 - 4x + 4)$			

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Exercise 3: How Well Did I understand?

Create a rational function with a vertical asymptote of x= -5 and a hole at x=4

Reflection: I learned in this topic that rational function is______________________

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References:

Oronce, Orlando. RBS General Mathematics First Edition

Learner's Material for Mathematics Grade 9

Note: Practice Personal Hygiene protocols at all times

Answer key

Activity 1: Know Me Better

1. Rational Function: Both the numerator and denominator are polynomials.
2. Rational Function: Both the numerator and denominator are polynomials.
3. Not a Rational Function: Numerator is not a polynomial
4. Rational Function: Denominator is not polynomial.

Activity 2: Place Me On the Table

	Zeros	x-and y-Intercepts	Asymptotes
1	X=5	x-intercept: (5,0)	Vertical Asymptote: x=-2
		y-intercept: (0,-5/2)	Horizontal Asymptote: y=1
2	The function f has no zero	x-intercept: (5, 0)	Vertical Asymptote: x=-5
		y-intercept: (0, 1/5)	Horizontal Asymptote: y=0
3	X=4 and x=1	x-intercept: (4,0) and (1,0)	Vertical Asymptote: x=2
		y-intercept: (0,1)	Horizontal Asymptote: y=1

Activity 3: How Well Did I understand?

There are many possible answers (Hint: Be sure that one factor in the numerator is x-4 and the denominator is (x+5) and (x-4)

Illustrations:

1. f(x)=$(x-4)(x+1)/(x+5)(x-4)$ *or* f(x)= $\frac{x^2-3x-4}{x^2+x-20}$
2. f(x)=$(x-4)(\_\_\_\_\_)/(x+5)(x-4)$ *or* f(x)= $\frac{x^2\_\_\_\_}{x^2+x-20}$
3. f(x)=$(x-4)(\_\_\_\_\_\_)/(x+5)(x-4)$ *or* f(x)= $\frac{x^2\_\_\_\_}{x^2+x-20}$