GENERAL MATHEMATICS 11

Name of Learner: ____________________ Grade Level: _________________

Section: ____________________________ Date: _______________________

LEARNING ACTIVITY SHEET

PERFORMS ADDITION, SUBTRACTION, MULTIPLICATION, DIVISION, AND COMPOSITION OF FUNCTIONS

Background Information for the Learners

OPERATIONS ON FUNCTIONS

Given two functions f and g, then:

Sum of f and g: $(f+g)(x) = f(x) + g(x)$

Difference of f and g: $(f-g)(x) = f(x) - g(x)$

Product of f and g: $(f \bullet g)(x) = f(x) \bullet g(x)$

Quotient of f and g: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, where $g(x) \neq 0$

Each of the four operations is defined for all x in the domains of both f and g, with the exception that in quotient, we exclude the values of x for which $g(x) = 0$.

Example 1.

Given the two functions, $f(x) = x^2 - 1$ and $g(x) = x^2 - x$. Compute $f(x) + g(x)$, $f(x) - g(x)$, $f(x) \bullet g(x)$ and $\frac{f(x)}{g(x)}$. Determine the domain of each operation.

Solution:

a. $f(x) + g(x) = (x^2 - 1) + (x^2 - x) = 2x^2 - x - 1,$ D: $\{x : x \in \mathbb{R}\}$

b. $f(x) - g(x) = (x^2 - 1) - (x^2 - x) = -1 + x,$ D: $\{x : x \in \mathbb{R}\}$

c. $f(x) \bullet g(x) = (x^2 - 1)(x^2 - x) = x^4 - x^3 - x^2 + x,$ D: $\{x : x \in \mathbb{R}\}$

d. $\frac{f(x)}{g(x)} = \frac{x^2-1}{x^2-x} = \frac{(x-1)(x+1)}{x(x-1)} = \frac{x+1}{x}$ D: $\{x : x \in \mathbb{R}, x \neq 0, 1\}$

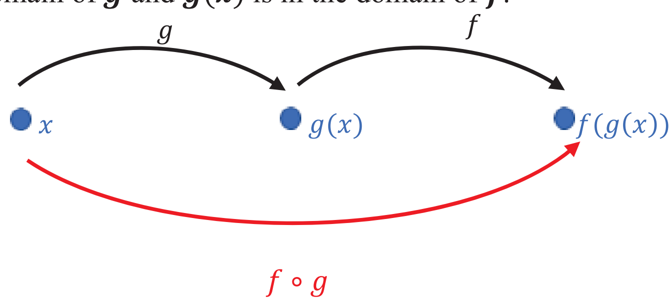
__Note: Practice Personal Hygiene protocols at all times__

THE COMPOSITE OF FUNCTIONS

The **composite** of f and g, denoted by $f \circ g$, is defined by two conditions:

1. $(\boldsymbol{f} \circ \boldsymbol{g})(\boldsymbol{x}) = \boldsymbol{f}(\boldsymbol{g}(\boldsymbol{x}))$**,** which is read as **"$\boldsymbol{f}$ circle $\boldsymbol{g}$ of $\boldsymbol{x}$ equals $\boldsymbol{f}$ of $\boldsymbol{g}$ of $\boldsymbol{x}$".**

2. $\boldsymbol{x}$ is in the domain of $\boldsymbol{g}$ and $\boldsymbol{g}(\boldsymbol{x})$ is in the domain of $\boldsymbol{f}$.



The domain of $f \circ g$ is the set of x satisfying condition (2). The operation that combines f *and* g to produce their composite is called **function composition**

Example 2.

Given $f(x) = x^2 + 3x - 4$ and $g(x) = x - 2$, evaluate the following.

a. $f(-1)$ b. $g(4)$ c. $f(g(x)$ d. $g(g(2)$ e. $(f \circ g)(2)$

Solution:

a. $f(-1) = (-1)^2 + 3(-1) - 4 = 6$

b. $g(4) = 4 - 2 = 2$

c. $f(g(x) = f(x - 2)$

$= (x - 2)^2 + 3(x - 2) - 4$

$= x^2 - 4x + 4 + 3x - 6 - 4$

$= x^2 - x - 6$

d. $g(g(2) = g(2 - 2)$

$= g(0)$

$= 0 - 2 = -2$

e. $(f \circ g)(2) = f[g(2)]$

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$= f[2-2]$

$= f[0]$

$= 0^2 + 3(0) - 4$

$= -4$

Example 3.

Given the functions, $f(x) = 2x - 1$, $g(x) = \sqrt{2x+1}$, $h(x) = \lfloor x \rfloor + 1$

a. Find and simplify $(g \circ f)(x)$.

b. Find and simplify $(h \circ g)(10)$

c. Find and simplify $h(2.1)[f(3) + g(4)]$

Solution:

a. $(g \circ f)(x) = g(f(x)) = \sqrt{2x - 1 + 1} = \sqrt{2x}$

b. $(h \circ g)(12) = h(g(10))$

$= \lfloor g(10) \rfloor + 1$

$= \left\lfloor \sqrt{2(10)+1} \right\rfloor + 1$

$= \lfloor \sqrt{21} \rfloor + 1$

$= 4 + 1$

$= 5$

c. $h(2.1)[f(3) + g(4)] = [\lfloor 2.1 \rfloor + 1][(2(3) - 1) + \sqrt{2(4)+1}]$

$= [2 + 1][5 + 3]$

$= 24$

Learning Competency

Performs addition, subtraction, multiplication, division, and composition of functions **(GM_M11GM-Ia-3)**

EXERCISE 1

Directions: Perform the indication operation in the following functions. **[1 point each]**

A. Given the functions $f(x) = 3x + 4$ and $g(x) = 3x^2$, find:

1. $(f + g)(x)$

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2. $(f - g)(x)$

3. $(fg)(x)$

4. $(f - g)(2)$

5. $(f + g)(-3)$

6. $\left(\frac{f}{g}\right)(x)$

B. Given $f(x) = x^2 + 1$, $g(x) = 2 - x$ and $q(x) = \frac{1}{x^2 + 4x - 3}$

7. $(f + g + q)(x)$

8. $(q - g)(-1)$

9. $(fgq)(0)$

10. $\left(\frac{fg}{q}\right)(2)$

EXERCISE 2

Directions: Solve the following functions **[2 points each]**

A. Let $f(x) = 2x + 4$, $g(x) = x^2 - 16$ and $h(x) = x^3$. Find:

1. $(f \circ g)(x)$

2. $(h \circ g)(x)$

3. $f(f\left(f\left(f(5)\right)\right))$

B. Let $f(x) = x^2 + 1$, $g(x) = \frac{1}{x}$ Find:

4. $(f \circ f)(x)$

5. $(g \circ g)(4)$

EXERCISE 3

Directions: Perform the indicated conditions in each function. **[2 points each]**

A. Let $f(x) = x^4$, $g(x) = \sqrt{x}$, $h(x) = x - 2$ and $p(x) = 3x$. Express each function t as a composite of three of these four functions.

1. $t(x) = 3(x - 2)^4$

2. $t(x) = (3x - 6)^4$

3. $t(x) = \sqrt{(x - 2)^4}$

4. $t(x) = \sqrt{x^4 - 2}$

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5. $t(x) = (3x)^2$

B. Let $f(x) = 2x - 3$, $g(x) = \frac{x+3}{2}$ and $h(x) = 3x + 2$ **[5 points each]**

1. Show that $f(g(x)) = g(f(x))$ for all x.

2. Show that $f\left(h(x)\right) \neq h\left(f(x)\right)$ for any x.

Reflection:

Please share your insights in this topic.

References:

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Alferez, G. S. (2014). *Introduction to Calculus*. Quezon City, Manila
Leithold, L. (1996). *The Calculus 7*. New York City.
Brown, R.G (1994*). Advanced Mathematics, Precalculus with Discrete Mathematics and Data Analysis*, Houghton Mifflin, Boston.
Rolando, M.A, et.al (2002). *Differential Calculus*. Philippines.

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ANSWER KEY

EXERCISE 1

A.

1. $3x^2 + 3x + 4$
2. $-3x^2 + 3x + 4$
3. $9x^3 + 12x^2$
4. -2
5. 22
6. $\frac{3x+4}{3x^2}$

B.

7. $x^2 - x + 3 + \left(\frac{1}{x^2+4x-3}\right)$
8. $-\frac{7}{6}$
9. $-\frac{2}{3}$
10. 0

EXERCISE 2

A.

1. $2x^2 - 28$
2. $x^6 - 48x^4 + 768x^2 - 4096$
3. 140

B.

1. $x^4 + 2x^2 + 2$
2. 4

EXERCISE 3

A.

1. $p\left(f\left(h(x)\right)\right)$
2. $f\left(p\left(h(x)\right)\right)$
3. $g\left(f\left(h(x)\right)\right)$
4. $g\left(h\left(f(x)\right)\right)$
5. $g\left(f\left(p(x)\right)\right)$

B.

1. $f\left(g(x)\right) = g(f(x) = x$
2. $f\left(h(x)\right) \neq h\left(f(x)\right) : 6x + 1 \neq 6x - 7$

Note: Practice Personal Hygiene protocols at all times