# GENERAL MATHEMATICS 11

Name of Learner:_____ Grade Level: _____
Section:_____ Score: _____

## Learning Activity Sheet
## Solving Logarithmic Equations & Logarithmic Inequalities

### Background Information for Learners

This activity sheet was crafted to serve as a learning guide for learners. It aims to improve the manipulative skills of learners specifically their skills in solving logarithmic equations and logarithmic inequalities.

In solving logarithmic equations and logarithmic inequalities, their properties should be taken into consideration.

*Property of Logarithmic Equations*
If b>1, then the logarithmic function $y = \log_b x$ is increasing for all x. If 0 < b < 1, then the logarithmic function $y = \log_b x$ is decreasing for all x. This means that $\log_b u = \log_b v$ if and only if u=v.

*Property of Logarithmic Inequalities*
If 0 < b < 1, then $x_1 < x_2$ if and only if $\log_b x_1 > \log_b x_2$
If b > 1, then $x_1 < x_2$ if and only if $\log_b x_1 < \log_b x_2$

There are also techniques in solving logarithmic equations and logarithmic inequalities.

*Techniques in Solving Logarithmic Equations*
- Rewriting the logarithmic equation to exponential form
- Using of logarithmic laws and properties
- Applying the one-to-one property of logarithmic functions
- Using the zero factor property

*Techniques in Solving Logarithmic Inequalities*
- Ensure that the logarithms are defined
- Make sure that the inequality is satisfied

Examine the given examples.

*Example 1:* Find the value of x in the following equations

a. $\log_8 4x = \log_8 24$
   Solution: $\log_8 4x = \log_8 24$
   4x = 24 (one-to-one property)
   x = 6

b. $\log_2(x-3) = 4$
   Solution: $\log_2(x-3) = 4$

$x - 3 = 2^4$ (Changing into exponential form)

$x - 3 = 16$

$x = 19$

*Example 2:* Solve the following logarithmic inequalities

a. $\log_4(x-2) > \log_4 8$

Solution: $\log_4(x-2) > \log_4 8$

$x - 2 > 8$ (Since b > 1, then $x_1 < x_2$ iff $\log_b x_1 < \log_b x_2$)

$x > 10$

Hence, the solution is $(10,+\infty)$

b. $\log_{\frac{1}{2}}(3x+4) \geq 4$

Solution: $\log_{\frac{1}{2}}(3x+4) \geq 4$

$\log_{\frac{1}{2}}(3x+4) \geq 4 \log_{\frac{1}{2}} \frac{1}{2}$ (Ensure that the logarithms are defined)

$\log_{\frac{1}{2}}(3x+4) \geq \log_{\frac{1}{2}}(\frac{1}{2})^4$

$\log_{\frac{1}{2}}(3x+4) \geq \log_{\frac{1}{2}} \frac{1}{16}$

$3x + 4 \leq \frac{1}{16}$ (since 0<b<1, then $x_1 < x_2$ iff $\log_b x_1 > \log_b x_2$)

$3x \leq \frac{-63}{16}$

$x \leq -\frac{21}{16}$

Thus, the solution is $(-\infty, -\frac{21}{16}]$

*Example 3:* A culture starts at 1,500 bacteria and doubles every 70 minutes. How long will it take the number of bacteria to reach 6,000?

Solution: A model for this situation is $y = 1{,}500\ (2)^{\frac{t}{70}}$, y is the number of bacteria at time t.

$6{,}000 = 1{,}500\ (2)^{\frac{t}{70}}$

$4 = (2)^{\frac{t}{70}}$

$ln\ 4 = ln(2)^{\frac{t}{70}}$

$ln\ 4 = \frac{t}{70} ln(2)$

$\frac{\ln 4}{\ln 2} = \frac{t}{70}$

$70\frac{\ln 4}{\ln 2} = t$

$70(2) = t$

$140 = t$

Therefore, it will take 140 minutes for the bacteria to reach 6,000.

## Learning Competency:

The learners solve logarithmic equations and inequalities **(M11GM-Ih-3)**

## Activity 1- Think and Strategize

Directions: Solve the following logarithmic equations. Specify the technique/s used.

16. $\log_3 4x = \log_3 24$

    Solution:

    Technique/s:

17. $\log_x 49=2$

    Solution:

    Technique/s:

18. $\log_2(8x)-\log_2(x-5)=4$

    Solution:

    Technique/s:

19. $(\log_4 x)^2 + 3 \log_4 x +2=0$

    Solution:

    Technique/s:

## Activity 2- Think, Think, Think Before You Solve

Directions: Solve the following logarithmic inequalities and write the solution in interval notation. Indicate the property used.

1. $\log_4 (3x+2) < \log_4 (x-4)$

   Property:

   Solution:

2. $\log_{2/3}(3x+1) > \log_{2/3}(5x+7)$

   Property:

   Solution:

3. $2 \geq \log_5(2x-3)$

Property:

Solution:

4. $-2 \leq \log x < 2$

   Property:

   Solution:

## Activity 3- Read, Analyze and Solve

Directions: Read, analyze and solve the following problems. Show you solutions.

1. Covid-19 pandemic has created apprehension to everyone. Many are hoping for the discovery of vaccine in order for this pandemic to come to an end. But since vaccine is not yet discovered, some avid individuals in social media immediately believe posted treatment of Covid-19 just to ease their apprehension. Suppose the equation of fake news on the treatment of Covid-19 pandemic is $p(t)= \frac{1}{1+15e^{-0.4t}}$, where p(t) is the proportion of the population who has believed of the fake news on the treatment of Covid-19 at time t days. How long will it take to reach the 90% of the population?

   Solution:

2. Assume that the culture of Covid-19 starts at 3,000 virus and triples every 60 minutes. How long will it take for the number of virus to reach 15,000?

   Solution:

3. Due to Covid-19 pandemic, many Filipinos are forced to lend money from institutions or individuals to suffice their basic needs. Suppose a lending institution uses the formula, $A=P(1+r)^n$ where A is the future value of the investment, P is the principal, r is the fixed annual interest rate and n is the number of years, how many years will it take a lender to pay his/her tripled debt if he/she borrowed P15,000 at a rate of 2.5% per annum?

   Solution:

## Rubrics for Scoring

Activity 1- Think And Strategize

| | 3 points | 2 points | 1 point | 0 point |
|---|---|---|---|---|
| Understanding | Learner shows a thorough understanding on how to solve logarithmic equations | Learner shows a partial understanding on how to solve logarithmic equations | Learner shows a limited understanding on how to solve logarithmic equations | Learner shows no understanding on how to solve logarithmic equations |
| Quantity of listed techniques | Learner listed all the possible techniques in solving a logarithmic equation | Learner listed some of the possible techniques in solving a logarithmic equation | Learner listed few possible techniques in solving a logarithmic equation | Learner did not write any technique |

Activity 2- Think, Think, Think Before you Solve

| | 3 points | 2 points | 1 point | 0 point |
|---|---|---|---|---|
| Understanding | Learner shows a thorough understanding on how to solve logarithmic inequalities | Learner shows a partial understanding on how to solve logarithmic inequalities | Learner shows a limited understanding on how to solve logarithmic inequalities | Learner shows no understanding on how to solve logarithmic inequalities |
| Additional 1 point will be given to the learner if he/she indicated the correct property. | | | | |

## Reflection

Share your feelings while doing the activity.

## References

Sharon L. Senk et al. UCSMP Advanced Algebra. Second Edition.(2002). Pearson Education Inc.

General Mathematics Learner's Material. First Edition 2016

# Answer Key

Activity 1- Think And Strategize

1. x=6; one-to-one property
2. x=7, -7 is not a solution; changing into exponential form & factorization/extracting the roots
3. x=10; using the law, $\log_b \frac{u}{v}$=$\log_b$u-$\log_b$v, & changing into exponential form
4. x=$\frac{1}{16}$ and x=$\frac{1}{4}$ ; factorization (using the zero factor property) & changing into exponential form

Activity 2- Think, Think, Think Before you Solve

1. Since b>1, then $x_1<x_2$ if and only if $\log_b x_1 < \log_b x_2$; x<-3, hence its solution is (-∞, -3)
2. Since 0<b<1, then $x_1<x_2$ if and only if $\log_b x_1 > \log_b x_2$; $x>-3$, hence its solution is $(-3, +\infty)$
3. Since b>1, then $x_1<x_2$ if and only if $\log_b x_1 < \log_b x_2$; x≤14, hence its solution is (-∞, 14]
4. Since b>1, then $x_1<x_2$ if and only if $\log_b x_1 < \log_b x_2$; $\frac{1}{100} < x < 100$, hence its solution is $[\frac{1}{100}, 100]$

Activity 3- Read, Analyze and Solve

1. $t \approx 12.26$
   It will take approximately 13 days for the fake news on the treatment of Covid-19 to reach the 90% of the population
2. $t \approx 87.90$
   It will take approximately 88 minutes or 1 hour & 28 minutes for the virus to reach 15,000
3. $n \approx 44.49$
   It will take approximately 45 years to pay the tripled debt.