GENERAL MATHEMATICS 11

Name of Learner:_____ Grade Level: _____

Section:_____ Date: _____

Learning Activity Sheet
Representing Real-Life Situations using Logarithms

Background Information for Learners

This activity sheet was designed to help learners understand the lesson on how to represent real-life situations using logarithmic functions. This would also enhance the learners' ability to solve. .

Logarithm is defined as $c=\log_b a$ if and only if $a=b^c$ where a,b and c are positive real numbers such that $b\neq1$. Meanwhile, common logarithm is a logarithm with base 10 and it is written as log x or $\log_{10}x$. On the other hand, natural logarithm is a logarithm to the base e (approximately 2.71828) and is denoted by ln x or $\log_e x$. Consider the examples below.

Example 1: Rewrite the following exponential equations in logarithmic form.
    a. $5^3=125$
        This can be written as $\log_5 125=3$ following the definition of logarithm
    b. $10^4=10,000$
        Since a= 10,000, b=10 and c=4, then its logarithmic form is $\log_{10} 10,000=4$ or log 10,000=4 because it is a common logarithm

Example 2: Rewrite the following logarithmic equations in exponential form.
    a. $\log_3 81=4$
        Its equivalent exponential form is $3^4=81$ since a=81, b=3 and c=4
    b. ln 10=x
        The given is an example of natural logarithm. Hence its exponential form is $e^x=10$.

There are common applications of logarithms in real-life. These are Richter scale, sound intensity and pH level. The formula of the magnitude (R) of an earthquake on a Richter scale is given by $R = \frac{2}{3}\log \frac{E}{10^{4.40}}$ where E is the energy released by the earthquake usually in joules while $10^{4.40}$ joules is the energy released by a very small reference earthquake. Meanwhile, the decibel level of sound is given by the formula, $D = 10 \log \frac{I}{10^{-12}}$ where I is the sound intensity in watts/m^2 and the quantity 10^{-12} watts/m^2 is the least audible sound a human can hear. For the pH level of a water-based solution, it is given by pH=-log[H^+] where H^+ is the concentration of hydrogen ions in moles per liter. Solutions with pH level of 7 are categorized as neutral, while those solutions with pH level less than 7 are acidic and those solutions with greater than 7 pH level are basic. Study the given examples.

Example 3: Suppose that an earthquake released approximately 10^8 joules of energy. What is its magnitude on a Richter scale? How much more energy does this earthquake release than the reference earthquake?

Solution:

Since the given is a sample problem of solving the magnitude of an earthquake in a Richter scale, then the formula to be used is $R = \frac{2}{3}\log\frac{E}{10^{4.40}}$. The value of E is given which is equal to 10^8 joules. Therefore, the formula becomes $R = \frac{2}{3}\log\frac{10^8}{10^{4.40}} = \frac{2}{3}\log 10^{8-4.40} = \frac{2}{3}\log 10^{3.6} = \frac{2}{3}(3.6) \approx 2.4$. Thus, the magnitude of the earthquake is approximately equal to 2.4

To compare the energy released by the given earthquake with the energy released by the reference earthquake, just divide the two energies. That is, $\frac{10^8}{10^{4.40}} = 10^{8-4.40} = 10^{3.6} \approx$ 3,981. This means that the earthquake releases approximately equal to 3,981 times more energy than the reference earthquake.

Example 4: The decibel level of sound in an office is 10^{-10} watts/m². What is the corresponding sound intensity in decibels? How much more intense is this sound than the least audible sound a human can hear?

Solution:

The formula to be used is $D = 10\log\frac{I}{10^{-12}}$. Since I is given which is 10^{-10} watts/m², then the working formula is $D = 10\log\frac{10^{-10}}{10^{-12}} = 10\log 10^{-10+12} = 10\log 10^2 = 10(2) = 20$. This means that the corresponding intensity is 20 decibels.

To compare the intensity of the two sounds, simplify $\frac{10^{-10}}{10^{-12}}$. This will yield to $10^{-10+12}= 10^2 = 100$ which means that the sound is 100 times more intense than the least audible sound a human can hear.

Example 5: A 1-liter solution contains 0.0000001 moles of hydrogen ion. Find its ph level.

Solution:

Since there are 0.0000001 moles of hydrogen ion, then its concentration is 10^{-7} moles per liter. Thus, the ph level is $-\log 10^{-7}=-(-7)=7$.

Learning Competency:
The learners represent real-life situations using logarithmic functions **(M11GM-Ih-1)**

Activity 1- Write What?
Directions: Read the following problems. Do as indicated. Write the answer before the item.

1. Rewrite the following exponential equations in logarithmic form whenever possible
    a. $4^3=64$
    b. $8^{-2}=\frac{1}{64}$
    c. $(\frac{1}{4})^2=\frac{1}{16}$
    d. $(-3)^2=9$
    e. $e^3=x$
2. Rewrite the following logarithmic equations in exponential form
    a. $\log c =d$

b. $\log_3 81 = 4$

c. $\log_{1/2} \frac{1}{32} = 5$

d. $\log 0.01 = -2$

e. $\ln 4 = x$

Activity 2- How are Logarithms Useful?

Directions: Read, analyze and solve the given problems. Show your solutions.

1. An earthquake has released 10^{15} joules of energy. Find its magnitude on a Richter scale?

   Solution:

2. In a certain room, its decibel level of sound is 10^{-8} watts/m^2. What is its corresponding sound intensity in decibels?

   Solution:

3. A solution contains 10^{-4} moles of hydrogen ions. Describe its pH level.

   Solution:

4. What is the magnitude of an earthquake on a Richter scale that released 10,000,000,000 joules of energy?

   Solution:

5. Find the pH level of a solution that contains 0.0000001 moles of hydrogen ions.

   Solution:

Activity 3- Fact or Bluff

Directions: Identify whether the following statements is correct or incorrect. Write Fact if it is correct and Bluff if it is incorrect. Explain your answer.

1. The magnitude in the Richter scale of a certain earthquake that released 10^9 joules of energy is approximately equal to 3.1.

   Answer:

2. In a 1-liter solution containing 10^{-9} moles of hydrogen ions, its pH level is 9. Hence, the solution is acidic.

   Answer:

3. The corresponding sound intensity in decibels of a jet during takeoff is 140 decibels. This means that the intensity of sound of a jet during takeoff is 1000 watts/m^2.

Answer:
4. A 1-liter solution becomes neutral when it contains 10^{-7} moles of hydrogen ions.
   Answer:

5. An earthquake that released an approximately 39,811 times more energy than the reference earthquake has 10^{10} joules of energy.

   Answer:

Rubrics for Scoring

Activity 1- Write What?

One point is given to the learner when he/she gets a correct answer

Activity 2- How are Logarithms Useful?

	3 points	2 points	1 point	0 point
Amount of work	Learner presented a complete solution with simplified answer	Learner presented an incomplete solution but simplified answer	Learner presented an incomplete solution	Learner presented nothing
Understanding	Learner shows a thorough understanding on the applications of logarithm in real-life	Learner shows a partial understanding on the applications of logarithm in real-life	Learner shows a limited understanding on the applications of logarithm in real-life	Learner shows no understanding on the applications of logarithm in real-life

Reflection
Construct at least three (3) sentences on the importance of logarithm in real-life.

References
Barnett, R.A, Ziegler, M.R., Byleen, K.E., &Sobecki,D.(2008).Precalculus.New York: McGraw-Hill
Young, C. (2012). College Algebra (3rd ed). Hoboken, NJ:John Wiley & Sons
Stewart, J., Redlin, L., & Watson, S. (2012). Pre-calculus: Mathematics for Calculus (6th ed).Belmont, CA: Brooks/Cole, Cengage Learning
General Mathematics Learner's Material

Answer Key

Activity 1- Write What?

1.a. $\log_4 64 = 3$

b. $\log_8 \frac{1}{64} = -2$

c. $\log 1/4 \frac{1}{16} = 2$

d. cannot be written in logarithmic form because b<1

e. $\log_e x = 3$ or $\ln x = 3$

2.a. $10^d = c$

b. $3^4 = 81$

c. $(\frac{1}{2})^5 = \frac{1}{32}$

d. $10^{-2} = 0.01$

e. $e^x = 4$

Activity 2- How are Logarithms Useful?

1. $R = \frac{2}{3}(10.6) \approx 7.1$
2. D=10(4)=40 decibels
3. pH= -(-4)=4→acidic
4. $R = \frac{2}{3}(5.6) \approx 3.7$
5. pH= -(-7)=7

Activity 3- Fact or Bluff

1. Fact because $R = \frac{2}{3}(4.6) \approx 3.1$
2. Bluff because 9>7, hence, it is basic
3. Bluff because 140≠ 10(15)
4. Fact because 7=neutral
5. Bluff because $\frac{10^{10}}{10^{4.40}} \neq 10^{4.6}$ or 39,811