GENERAL MATHEMATICS 11

Name of Learner: ______________________________ **Grade Level:** ________
Section: ______________________________ **Score:** ____________

LEARNING ACTIVITY SHEET

ILLUSTRATE DIFFERENT TYPES OF TAUTOLOGIES AND FALLACIES AND DETERMINE THE VALIDITY OF CATEGORICAL SYLLOGISMS

Background Information for Learners

A **valid argument** satisfies the validity condition; that is, the conclusion q is true whenever the premises $p_1, p_2, \ldots, p_n$ are all true. The argument is valid if the conditional $(p_1 \wedge p_2 \wedge \ldots \wedge p_n) \rightarrow q$ is a **tautology**.

An argument $(p_1 \wedge p_2 \wedge \ldots \wedge p_n) \rightarrow q$, which is not valid is called a **fallacy**.

Example 1

Prove that the argument $((p \rightarrow q) \wedge p) \rightarrow q$ known as **Modus Ponens** is valid.

Solution

Show that $((p \rightarrow q) \wedge p) \rightarrow q$ is a tautology.

p	Q	p → q	(p → q) ∧ p	(p → q) ∧ p → q
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Since $((p \rightarrow q) \wedge p) \rightarrow q$ is a tautology, then the argument is valid.

Example 2

Prove that the argument $((p \rightarrow q) \wedge q \rightarrow p$ is a fallacy. This is known as the **Fallacy of the Converse.**

Solution

Show that $((p \rightarrow q) \wedge q \rightarrow p$ is not a tautology using the truth table.

p	Q	p → q	(p → q) ∧ q	((p → q) ∧ q → p
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

On the third row, the premises q and p → q are both true but the conclusion p is false. The given argument is a fallacy.

Example 3
Determine whether the argument is valid.

If triangle T_1 and T_2 are congruent, then they are similar. Triangles T_1 and T_2 are congruent. Therefore, triangles T_1 and T_2 are similar.

Solution
The argument is valid by Modus Ponens. Furthermore, we know that from the geometry of triangles that congruent triangles are also similar (but similar are not necessary congruent).

Learning Competency with code

The learner is able to illustrate different types of tautologies and fallacies and determine the validity of categorical syllogisms (**M11GM-IIi-1-2,Quarter II**)

Directions/Instructions:

A**.** Complete the truth table for the given statement to show that the compound statement is a tautology.

1. $p \rightarrow (p \vee q)$

p	**q**	**$p \vee q$**	**$p \rightarrow (p \vee q)$**
T	T		
T	F		
F	T		
F	F		

2. $p \rightarrow (q \vee p)$

p	**Q**	**$q \vee p$**	**$p \rightarrow (q \vee p)$**
T	T		
T	F		
F	T		
F	F		

3. $(p \to q) \leftrightarrow (\sim q \to \sim p)$

p	q	p → q	~ q	~ p	~ q → ~ p	(p → q) ↔ (~ q → ~ p)
T	T					
T	F					
F	T					
F	F					

4. $(p \vee (\sim p)) \wedge (q \vee (\sim q))$

p	q	~ p	p ∨ ~ p	~ q	q ∨ ~ q	(p ∨ (~ p)) ∧ (q ∨ (~ q))
T	T					
T	F					
F	T					
F	F					

5. $\sim (p \wedge q) \leftrightarrow (\sim p \vee \sim q)$

p	Q	p ∧ q	~ (p ∧ q)	~ p	~ q	~ p ∨ ~ q	~ (p ∧ q) ↔ (~ p ∨ ~ q)
T	T						
T	F						
F	T						
F	F						

B. Determine whether the symbolic form of the argument is valid or a fallacy using a truth table.

1. $\begin{array}{l} p \to q \\ \underline{p} \\ \therefore p \end{array}$

2. $\begin{array}{l} \underline{p \vee q} \\ \therefore p \wedge q \end{array}$

3. $\begin{array}{l} p \vee \sim q \\ \underline{\sim q} \\ \therefore \sim p \end{array}$

4. $\begin{array}{l} \sim p \wedge q \\ \underline{p \leftrightarrow r} \\ \therefore p \wedge r \end{array}$

5. $\begin{array}{l} p \to q \\ \underline{q \to r} \\ \therefore \sim q \to \sim r \end{array}$

C. Determine whether the following arguments are valid. If it is valid, identify the rule of inference which justifies the validity otherwise identify the type of fallacy exhibited by the argument.

1. All tigers are mammals.
   No mammals are creatures with scales.
   Therefore, no tigers are creatures with scales

2. All spider monkeys are elephants.
   No elephants are animals.
   Therefore, no spider monkeys are animals

3. Today isn't a holiday.
   If there will be mail delivery, then today isn't a holiday.
   Therefore, there will be mail delivery.

4. If today is Tuesday, then I have to finish my homework.
   If I have to finish my homework, then I have to go to work.
   Therefore, if today is Tuesday, then I have to go work.

5. If you drive a BMW, then you are a telemarketer
   If you are a telemarketer, then you are rolling in cash.
   Therefore, if you drive a BMW, then you are rolling in cash.

6. You sell used cars, or you are charming.
   You don't sell used cars.
   Therefore, you are charming.

7. If you will bring me a cake, then today is my birthday.
   If today is my birthday, then you will send flowers.
   Therefore, if you will bring me a cake, then you will send flowers.

8. Today is Monday, or life doesn't look bleak.
   Today is Monday.
   Therefore, life looks bleak.

9. You floss twice a day, or you don't brush after every meal.
   You floss twice a day.
   Therefore, you brush after every meal.

10. If today is a holiday, then we'll have a picnic.
    Today is holiday.
    Therefore, we'll have a picnic.

Reflection

What I have learned in this activity

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References:

Department of Education General Mathematics (Teacher's Guide).2016

Orines, Fernando B. *Next Century Mathematics (General Mathematics)*.Phoenix Publishing House, Inc.2016

Oronce, Orlando A. *General Mathematics*. Rex Book Store.2016

http://www.math.fsu.edu

Answer Key

A

1. $p \rightarrow (p \vee q)$

p	q	p ∨ q	p → (p ∨ q)
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

2. $p \rightarrow (q \vee p)$

p	q	q ∨ p	p → (q ∨ p)
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

3. $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$

p	q	p → q	~ q	~ p	~ q → ~ p	(p → q) ↔ (~ q → ~ p)
T	T	T	F	F	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

4. $(p \vee (\sim p)) \wedge (q \vee (\sim q))$

p	q	~ p	p ∨ ~ p	~ q	q ∨ ~ q	(p ∨ (~ p)) ∧ (q ∨ (~ q))
T	T	F	T	F	T	T
T	F	F	T	T	T	T
F	T	T	T	F	T	T
F	F	T	T	T	T	T

5. $\sim (p \wedge q) \leftrightarrow (\sim p \vee \sim q)$

p	q	p ∧ q	~ (p ∧ q)	~ p	~ q	~ p ∨ ~ q	~ (p ∧ q) ↔ (~ p ∨ ~ q)
T	T	T	F	F	F	F	T
T	F	F	T	F	T	T	T
F	T	F	T	T	F	T	T
F	F	F	T	T	T	T	T

B

1. The symbolic statement is $((p \rightarrow q) \wedge p) \rightarrow p$

p	q	p → q	p → q) ∧ p	((p → q) ∧ p) → p
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

The entries in the final column of the truth table are all true, so the argument is valid.

2. The symbolic statement is $(p \vee q) \rightarrow (p \wedge q)$

p	q	p ∨ q	p ∧ q	(p ∨ q) → (p ∧ q)
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

The entries in the final column of the truth table are not all true, so the argument is a fallacy.

3. The symbolic statement is $((p \vee \sim q) \wedge \sim q) \rightarrow \sim p$

p	q	~ q	p ∨ ~ q	(p ∨ ~q) ∧ ~ q	~ p	((p ∨ ~q) ∧ ~ q) → ~ p
T	T	F	T	F	F	T
T	F	T	T	T	F	F
F	T	F	F	F	T	T
F	F	T	T	T	T	T

The entries in the final column of the truth table are not all true, so the argument is a fallacy.

4. The symbolic statement is $(\sim p \wedge q) \wedge (p \leftrightarrow r) \rightarrow (p \wedge r)$

p	q	r	~ p	~p ∧ q	p ↔ r	(~p ∧ q) ∧ (p ↔ r)	p ∧ r	(~p ∧ q) ∧ (p ↔ r) → (p ∧ r)
T	T	T	F	F	T	F	T	T
T	T	F	F	F	F	F	F	T
T	F	T	F	F	T	F	T	T
T	F	F	F	F	F	F	F	T
F	T	T	T	T	F	F	F	T
F	T	F	T	T	T	T	F	F
F	F	T	T	F	F	F	F	T
F	F	F	T	F	T	F	F	T

The entries in the final column of the truth table are not all true, so the argument is a fallacy.

5. The symbolic statement is $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (\sim q \rightarrow \sim r)$

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$(p \rightarrow q) \wedge (q \rightarrow r)$	~ q	~ r	$\sim q \rightarrow \sim r$	$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (\sim q \rightarrow \sim r)$
T	T	T	T	T	T	F	F	T	T
T	T	F	T	F	F	F	T	T	T
T	F	T	F	T	F	T	F	F	T
T	F	F	F	T	F	T	T	T	T
F	T	T	T	T	T	F	F	T	T
F	T	F	T	F	F	F	T	T	T
F	F	T	T	T	T	T	F	F	F
F	F	F	T	T	T	T	T	T	T

The entries in the final column of the truth table are not all true, so the argument is a fallacy.

C

1. Valid (Modus Tollens)
2. Valid (Modus Tollens)
3. Invalid (Fallacy Of The Converse)
4. Valid (Rule Of Disjunctive Syllogism)
5. Valid (Law Of Syllogism)
6. Valid (Rule Of Disjunctive Syllogism)
7. Valid (Law Of Syllogism)
8. Invalid (Affirming The Disjunct)
9. Invalid (Affirming The Disjunct)
10. Valid (Modus Ponens)