

GENERAL MATHEMATICS 11

Name of Learner: _____
Section: _____

Grade Level: _____
Date: _____

LEARNING ACTIVITY SHEET

Solving Exponential Equations and Inequalities

Background Information for Learners

In solving **exponential equation**, remember the property of equality for exponential functions that if the base b is a positive number other than 1, then $b^x = b^y$ if and only if $x = y$.

Steps in Solving Exponential Equation

- Rewrite both sides of the equation using the same base.
- Remove the bases and set the exponents equal to each other.
- Simplify and solve for the x variable.

Example 1. $2^x = 64$
 $2^x = 2^6$
 $x = 6$

On the other hand, an **exponential inequality** is an inequality involving exponential expressions. It is very important to consider the properties of exponential inequalities which are indicated on the table.

If $b > 1$

then $b^x > b^y$ if and only if $x > y$

and $b^x < b^y$ if and only if $x < y$

If $0 < b < 1$

then $b^x > b^y$ if and only if $x < y$

and $b^x < b^y$ if and only if $x > y$

Steps in solving exponential inequality

- Rewrite the inequality using the same base.
- Remove the bases and set the exponents.
- Reverse the inequality sign if the base is greater 0 but less than 1. If the base is greater than 1, the direction of inequality sign is retained.
- Simplify and solve for the x variable.

Example 2. $4^{3x+2} < 64$
 $4^{3x+2} < 4^3$
 $3x + 2 < 3$
 $3x < 1$
 $x < \frac{1}{3}$

Solution: $x < \frac{1}{3}$

Interval Notation: $(-\infty, \frac{1}{3})$

Example 3. $0.3^{3x-2} < 0.3^{x+2}$
 $3x - 2 > x + 2$
 $2x > 4$
 $x > 2$

Learning Competency with code

The learner is able to solve exponential equations and inequalities. M11GM-Ie-f-1

Directions

Below are the solved examples of exponential equation and inequalities, which will be your basis in answering the exercises.

Solve $9^{x-3} = 3^{x+7}$.

$$\begin{aligned} 9^{x-3} &= 3^{x+7} \\ (3)^{2(x-3)} &= 3^{x+7} \\ 2(x-3) &= x+7 \\ 2x-6 &= x+7 \\ x &= 13 \end{aligned}$$

Solve $2^{3x+4} > \frac{1}{32}$

$$\begin{aligned} 2^{3x+4} &> \frac{1}{32} \\ 2^{3x+4} &> \frac{1}{2^5} \\ 2^{3x+4} &> 2^{-5} \\ 3x+4 &> -5 \\ 3x &> -9 \\ x &> -3 \end{aligned}$$

Solve $\left(\frac{1}{2}\right)^{3x+1} > \left(\frac{1}{4}\right)^{x+2}$

$$\begin{aligned} \left(\frac{1}{2}\right)^{3x+1} &> \left(\frac{1}{4}\right)^{x+2} \\ \left(\frac{1}{2}\right)^{3x+1} &> \left(\frac{1}{2^2}\right)^{x+2} \\ \left(\frac{1}{2}\right)^{3x+1} &> \left(\frac{1}{2}\right)^{2(x+2)} \\ \left(\frac{1}{2}\right)^{3x+1} &> \left(\frac{1}{2}\right)^{2x+4} \\ 3x+1 &< 2x+4 \\ x &< 3 \end{aligned}$$

EXERCISE 1. “MAKE AN ANALYSIS”

Directions: Make an analysis on the solutions presented in each item.

1. Ligaya and Mutya are solving $4^{2x} = 2^{x+6}$.

LIGAYA’S SOLUTION

$$\begin{aligned} 4^{2x} &= 2^{x+6} \\ 2^{2(2x)} &= 2^{x+6} \\ 4x &= x+6 \\ 3x &= 6 \\ x &= 2 \end{aligned}$$

MUTYA’S SOLUTION

$$\begin{aligned} 4^{2x} &= 2^{x+6} \\ 2x &= x+6 \\ 2x-x &= x+6-x \\ x &= 6 \end{aligned}$$

Question: Did anyone arrived with the correct answer? If there is, identify whose solution is right and explain why his/her solution is the correct one.

2. Mario and Antonio are solving $\frac{1}{64}^{x+3} = 8^{x-5}$.

MARIO’S SOLUTION

$$\begin{aligned} \frac{1}{64}^{x+3} &= 8^{x-5} \\ \frac{1}{2^6}^{x+3} &= 2^{3(x-5)} \\ 2^{6(x+3)} &= 2^{3x-15} \\ 6x+18 &= 3x-15 \\ 3x &= -33 \\ x &= -11 \end{aligned}$$

ANTONIO’S SOLUTION

$$\begin{aligned} \frac{1}{64}^{x+3} &= 8^{x-5} \\ \frac{1}{2^6}^{x+3} &= 2^{3(x-5)} \\ 2^{-6(x+3)} &= 2^{3x-15} \\ -6x-18 &= 3x-15 \\ -3 &= 9x \\ x &= -\frac{1}{3} \end{aligned}$$

Question: Did anyone arrived with the correct answer? If there is, identify whose solution is right and explain why his/her solution is the correct one.

3. Lorie and Daniel are solving $0.5^{3x+5} \geq 0.25^{x-2}$.

LORIE'S SOLUTION

$$0.5^{3x+5} \geq 0.25^{x-2}$$

$$0.5^{3x+5} \geq 0.5^{2(x-2)}$$

$$3x + 5 \geq 2x - 4$$

$$x \geq -9$$

DANIEL'S SOLUTION

$$0.5^{3x+5} \geq 0.25^{x-2}$$

$$0.5^{3x+5} \geq 0.5^{2(x-2)}$$

$$3x + 5 \leq 2x - 4$$

$$x \leq -9$$

Question: Did anyone arrived with the correct answer? If there is, identify whose solution is right and explain why his/her solution is the correct one.

4. Eva and Dave are solving $\left(\frac{3}{5}\right)^{2x-2} \geq \left(\frac{125}{27}\right)^{x+2}$.

EVAS'S SOLUTION

$$\left(\frac{3}{5}\right)^{2x-2} \geq \left(\frac{125}{27}\right)^{x+2}$$

$$\left(\frac{3}{5}\right)^{2x-2} \geq \left(\frac{5}{3}\right)^{3(x+2)}$$

$$\left(\frac{3}{5}\right)^{2x-2} \geq \left(\frac{3}{5}\right)^{-3(x+2)}$$

$$2x - 2 \geq -3x - 6$$

$$-\frac{4}{5} \geq x$$

DAVE'S SOLUTION

$$\left(\frac{3}{5}\right)^{2x-2} \geq \left(\frac{125}{27}\right)^{x+2}$$

$$\left(\frac{3}{5}\right)^{2x-2} \geq \left(\frac{5}{3}\right)^{3(x+2)}$$

$$\left(\frac{3}{5}\right)^{2x-2} \geq \left(\frac{3}{5}\right)^{-3(x+2)}$$

$$2x - 2 \leq -3x - 6$$

$$\frac{4}{5} \leq x$$

Question: Did anyone arrived with the correct answer? If there is, identify whose solution is right and explain why his/her solution is the correct one.

EXERCISE 2. "HOW DID YOU GET ME?"

Directions: Solve each of the following exponential equations and inequalities. Your task is to show the step-by-step process on how it arrive with the given answer.

1. $625^{x-1} = 25^{x+5}$

2. $\left(\frac{1}{6}\right)^{2x} = 216^{x-5}$

Answer: $x = 7$

3. $0.4^{3x+2} > 0.064^{3-2x}$

Answer: $x = 3$

4. $\left(\frac{1}{49}\right)^{3x-3} \leq 343^{x+2}$

Answer: $x < \frac{7}{9}$

Answer: $x \geq 0$

EXERCISE 3. "SOLVE ME"

A. Solve the following exponential equation and inequalities.

[3 points each]

1. $9^{3x-8} = \left(\frac{1}{81}\right)^{x-2}$

Hint: $\frac{1}{81}$ can be written as $\frac{1}{9^2}$ or 9^{-2}

2. $16^{2x-4} \geq 32^{x-1}$

3. $\frac{1}{343}^{x+2} = \left(\frac{1}{49}\right)^{x-4}$

4. $27^{x+3} \geq \left(\frac{1}{9}\right)^{2x+4}$

B. Solve the following worded problems involving exponential equations.

[3 points each]

1. At $t = 0$, there were initially 960 grams of an isotope. The half-life of an isotope is 30 hours. How much time has passed when only 60 grams of the isotope were left?
2. It is 8 o'clock in the morning when a scientist starts examining 25 samples of germs that triples every 4 hours. What time of the day will he found out that the initial 25 germs had already grown to 675?

Rubric for Scoring

For Exercise 1

CRITERIA	4 POINTS	3 POINTS	2 POINTS	1 POINT
Explanation of solution	Explanation clearly indicates procedures are understood at an advanced level	Explanation indicates procedures are understood	Explanation indicates a minimal understanding of procedure	Explanation is present but demonstrates a lack of understanding

POSSIBLE TOTAL POINTS: 16

For Exercise 2

CRITERIA	4 POINTS	3 POINTS	2 POINTS	1 POINT
Equation Solution	Initial equation and all steps are shown	Initial equation and most of the steps are shown.	Incorrect solution Arrived at the correct/incorrect answer	Partially started showing the solution, but did not finish it.

POSSIBLE TOTAL POINTS: 16

Reflection

What have you learned from this topic and how can you relate it to real life situations?

References for Learners

Verzosa, D. et. Al. (2016). General Mathematics. Lexicon Press Inc.

<https://www.mathwarehouse.com/algebra/exponents/solve-exponential-equations-how-to.php>

<https://www.mathemania.com/lesson/exponential-inequalities/>

Answer Key

EXERCISE 1

1. Ligaya is correct because she rewrite the equation with the same base before setting the exponents equal to each other. Mutya's solution is incorrect. Mutya should rewrite both sides of the equations as powers of the same base before computing the value of x. The correct answer is $x = 2$, which was obtained by Ligaya.
2. Antonio is correct and Mario is not. Antonio used the law of negative exponent which Mario failed to do when he transformed $\frac{1}{2^6}^{x+3} = 2^{3(x-5)}$ to $2^{6(x+3)} = 2^{3x-15}$. The correct answer was obtained by Antonio which is $x = -\frac{1}{3}$.

3. Daniel is correct and Lorie is not. It is right to reverse the direction of inequality sign since the base is less than 1. The correct answer is $x \leq -9$, which was attained by Daniel.
4. No one got the correct answer. Eva should have reversed the direction of the inequality sign. Dave reversed the inequality sign but failed in simplifying his answer. The correct solution for the inequality $\left(\frac{3}{5}\right)^{2x-2} \geq \left(\frac{125}{27}\right)^{x+2}$ is $x \geq -\frac{4}{5}$.

EXERCISE 2

1. $625^{x-1} = 25^{x+5}$

$$5^{3(x-1)} = 5^{2(x+5)}$$

$$3(x-1) = 2(x+5)$$

$$3x - 3 = 2x + 10$$

$$x = 7$$

Answer: $x = 7$

3. $0.4^{3x+2} > 0.064^{3-2x}$

$$0.4^{3x+2} > 0.4^{3(3-2x)}$$

$$3x + 2 < 3(3 - 2x)$$

$$3x + 2 < 9 - 6x$$

$$9x < 7$$

$$x < \frac{7}{9}$$

Answer: $x < \frac{7}{9}$

2. $\left(\frac{1}{6}\right)^{2x} = 216^{x-5}$

$$6^{-1(2x)} = 6^{3(x-5)} \quad \text{or} \quad \left(\frac{1}{6}\right)^{2x} = \left(\frac{1}{6}\right)^{-3(x-5)}$$

$$-1(2x) = 3(x-5) \quad 2x = -3(x-5)$$

$$-2x = 3x - 15 \quad 2x = -3x + 15$$

$$x = 3 \quad x = 3$$

Answer: $x = 3$

4. $\left(\frac{1}{49}\right)^{3x-3} \leq 343^{x+2}$

$$\left(\frac{1}{7^2}\right)^{3x-3} \leq 7^{3(x+2)}$$

$$7^{-2(3x-3)} \leq 7^{3(x+2)}$$

$$-2(3x-3) \leq 3(x+2)$$

$$-6x + 6 \leq 3x + 6$$

$$0 \leq 9x$$

$$0 \leq x$$

Answer: $x \geq 0$

EXERCISE 3

A.

1. $x = 2\frac{2}{5}$ or 2.4
2. $x \geq \frac{11}{3}$ or $3\frac{2}{3}$ or 3.6
3. $x = -14$
4. $x \geq -\frac{17}{7}$ or -2.42857

B.

1. $t = 120$ hours
2. $t = 12$ hours, therefore, the germs had grown to 675 at 8 o'clock pm.