GENERAL MATHEMATICS 11

Name of Learner:______________________ Grade Level:________

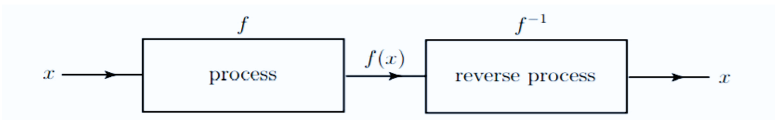
Section:______________________________ Score:_____________

LEARNING ACTIVITY SHEET
INVERSE OF ONE-TO-ONE FUNCTIONS

Background Information for Learners

We have learned that a function can be regarded as taking an input, x, and processing it in some way to produce a single output f(x). This time, we will find another function that will start with f(x) and process it to produce x again.

> ***Definition***
>
> Let $\boldsymbol{f}$ be a one-to-one function with domain $\boldsymbol{A}$ and range $\boldsymbol{B}$. Then the inverse of $\boldsymbol{f}$, denoted $\boldsymbol{f^{-1}}$, is a function with domain $\boldsymbol{B}$ and range $\boldsymbol{A}$ defined by $\boldsymbol{f^{-1}(y) = x}$ if and only if $\boldsymbol{f(x) = y}$ for any $\boldsymbol{y}$ in $\boldsymbol{B}$.



$figure$ 1. f^{-1} *reverses the process in* f

Note: A function has an inverse if and only if it is one-to-one function.

Finding the inverse of a one-to-one function

1. Write the function in the form $\boldsymbol{y = f(x)}$;
2. Interchange $\boldsymbol{x}$ and $\boldsymbol{y}$ variables;
3. Solve for $\boldsymbol{y}$ in terms of $\boldsymbol{x}$. The resulting equation is $\boldsymbol{f^{-1}(x)}$**.**

Example 1. Find the inverse of $\boldsymbol{f(x) = 8x + 5}$

Solution.

$y = f(x)$	$f(x) = 8x + 5$ ➡	$y = 8x + 5$
Interchange $\boldsymbol{x}$ and $\boldsymbol{y}$ variables	$y = 8x + 5$ ➡	$x = 8y + 5$
Solve for $\boldsymbol{y}$ in terms of $\boldsymbol{x}$		$x = 8y + 5$
		$x - 5 = 8y$

Note: Practice Personal Hygiene protocols at all times

$$\frac{x-5}{8} = y \Rightarrow y = \frac{x-5}{8}.$$

Therefore, the inverse of $f(x) = 8x + 5$ is $f^{-1}(x) = \frac{x-5}{8}$.

Check whether $f(x) = 8x + 5$ and $f^{-1}(x) = \frac{x-5}{8}$ are inverses.

suppose x=3 in $f(x)$.

$f(x) = 8x + 5 \Rightarrow \boldsymbol{f(3) = 8(3) + 5 = 29}$

$f^{-1}(x) = \frac{x-5}{8} \Rightarrow \boldsymbol{f^{-1}(29) = \frac{x-5}{8} = 3}$

Domain of f = Range of f^{-1} **Range of f = Domain of f^{-1}**

Hence, the two functions are inverses of each other.

Learning Competency

The learner is able to determine the inverse of a one-to-one function. ***M11GM-Id-2***

Exercise 1.

State if the given functions are inverses. (2 points each)

1. $f(x) = \frac{x-5}{10}$
   $g(x) = 10x + 5$
2. $f(x) = \frac{8+9x}{2}$
   $g(x) = \frac{5x-9}{2}$
3. $f(x) = \sqrt[5]{\frac{x-1}{2}}$
   $g(x) = 2x^5 + 1$
4. $f(x) = \frac{2}{x+3}$
   $g(x) = \frac{3x+2}{x+2}$
5. $f(x) = \frac{4-x}{x}$
   $g(x) = \frac{4}{x}$
6. $f(x) = \frac{-2-2x}{x}$
   $g(x) = \frac{-2}{x+2}$
7. $f(x) = 4 - \frac{3}{2}x$
   $g(x) = \frac{1}{2}x + \frac{3}{2}$
8. $f(x) = 4x + 16$
   $g(x) = \frac{-16+x}{4}$
9. $f(x) = 3x + 1$
   $g(x) = 3 + x$
10. $f(x) = -\frac{2}{x} - 1$
    $g(x) = \frac{-2}{x+1}$

Note: Practice Personal Hygiene protocols at all times

Exercise 2.

Find the inverse of each function. (4 points each)

1. $f(x) = x - 6$

2. $g(x) = -5x + 1$

3. $h(x) = \frac{4}{x+2}$

4. $f(x) = (x + 3)^3$

5. $g(x) = 2x^3 + 3$

6. $f(x) = \frac{3}{x-4}$

Note: Practice Personal Hygiene protocols at all times

7. $g(x) = \frac{-2x+1}{3}$

8. $h(x) = \frac{7-3x}{x-2}$

9. $f(x) = \sqrt[5]{\frac{-x+2}{2}}$

10. $g(x) = \frac{4}{5}x - 4$

Exercise 3.

Identify whether the inverse of each function is a function or not. Explain briefly your answer. (4 points each)

1. $f(x) = \frac{1}{2}x + 1$
2. $f(x) = x^2 + 6x + 9$
3. $f(x) = |2x|$

Reflection

What significant learnings have you acquired in this lesson?

____
____
____
____

Reference for Learners

Verzosa, D.B, et.al (2016). General Mathematics. Quezon City, Manila
https://cdn.kutasoftware.com/Worksheets/Alg2/Function%20Inverses.pdf

Note: Practice Personal Hygiene protocols at all times

Answer Key

Exercise 1.

1. $f(x) = \frac{x-5}{10}$
   $g(x) = 10x + 5$ YES

2. $f(x) = \frac{8+9x}{2}$
   $g(x) = \frac{5x-9}{2}$ NO
   $f^{-1}(x) = \dfrac{2x-8}{9}$

3. $f(x) = \sqrt[5]{\frac{x-1}{2}}$
   $g(x) = 2x^5 + 1$ YES

4. $f(x) = \frac{2}{x+3}$
   $g(x) = \frac{3x+2}{x+2}$ NO
   $f^{-1}(x) = \dfrac{2-3x}{x}$

5. $f(x) = \frac{4-x}{x}$
   $g(x) = \frac{4}{x}$ NO
   $f^{-1}(x) = \dfrac{4}{x+1}$

6. $f(x) = \frac{-2-2x}{x}$
   $g(x) = \frac{-2}{x+2}$ YES

7. $f(x) = 4 - \dfrac{3}{2}x$
   $g(x) = \frac{1}{2}x + \frac{3}{2}$ NO
   $f^{-1}(x) = \dfrac{8-2x}{3}$

8. $f(x) = 4x + 16$
   $g(x) = \frac{-16+x}{4}$ YES

9. $f(x) = 3x + 1$
   $g(x) = 3 + x^3$ NO
   $f^{-1}(x) = \dfrac{x-1}{3}$

10. $f(x) = -\frac{2}{x} - 1$
    $g(x) = \frac{-2}{x+1}$ YES

Exercise 2.

1. $f(x) = x - 6$
   $f^{-1}(x) = x + 6$

2. $g(x) = -5x + 1$
   $g^{-1}(x) = \dfrac{-x+1}{5}$

3. $h(x) = \frac{4}{x+2}$
   $h^{-1}(x) = \dfrac{4-2x}{x}$

4. $f(x) = (x+3)^3$
   $f^{-1}(x) = \sqrt[3]{x} - 3$

Note: Practice Personal Hygiene protocols at all times

5. $g(x) = 2x^3 + 3$

$$g^{-1}(x) = \sqrt[3]{\frac{x-3}{2}}$$

6. $f(x) = \frac{3}{x-4}$

$$f^{-1}(x) = \frac{4x+3}{x}$$

7. $g(x) = \frac{-2x+1}{3}$

$$g^{-1}(x) = \frac{-3x+1}{2}$$

8. $h(x) = \frac{7-3x}{x-2}$

$$h^{-1}(x) = \frac{2x+7}{x+3}$$

9. $f(x) = \sqrt[5]{\frac{-x+2}{2}}$

$$f^{-1}(x) = -2x^5 + 2$$

10. $g(x) = \frac{4}{5}x - 4$

$$g^{-1}(x) = 5 + \frac{5}{4}x$$

Exercise 3.

1. $f(x) = \frac{1}{2}x + 1$
   $f^{-1}(x) = 2x - 2$
   The given function is one-to-one function; thus, its inverse is also a one-to-one function.

2. $f(x) = x^2 + 6x + 9$
   The given function is a quadratic function; thus, it is not a one-to-one function. Therefore, the function $f(x) = x^2 + 6x + 9$ has no inverse function.

   Or

   f(x)= x^2+6X+9
   y= x^2+6X+9
   x=y^2+6y+9
   x-9= y^2+6y
   x= y^2+6y+9
   x=$(y+3)^2$
   $\pm\sqrt{x} = y + 3$
   $y = \pm\sqrt{x} - 3$

   The equation does not represent a function because there are some x-value that correspond to two different y-values (if x=4; y can be -1 or -5). Therefore, the function f(x)= x^2+6X+9 has no inverse function.

3. $f(x) = |2x|$
   y= $|2x|$
   x= $|2y|$
   x= $\sqrt{(2y)^2}$ ⟹ $|x| = \sqrt{x^2}$

   $\frac{x^2}{2} = y^2$
   $y = \pm\sqrt{\frac{x^2}{2}}$; This is not a function, therefore, $f(x) = |2x|$, has no inverse function.