11

General Mathematics

Quarter 1 – Module 4:

Inverse Functions

SELF-LEARNING MODULE

DEPARTMENT OF EDUCATION - SOCCSKSARGEN

General Mathematics – Grade 11
Self-Learning Module (SLM)
Quarter 1 – Module 4: Inverse Functions
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Introductory Message

For the facilitator:

Welcome to the Grade 11 General Mathematics Self-Learning Module (SLM) on Inverse Functions!

This module was collaboratively designed, developed and reviewed by educators both from public and private institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This learning resource hopes to engage the learners into guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:

Notes to the Teacher

This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.

For the learner:

Welcome to the Grade 11 General Mathematics Self-Learning Module (SLM) on Inverse Functions!

The hand is one of the most symbolized part of the human body. It is often used to depict skill, action and purpose. Through our hands we may learn, create and accomplish. Hence, the hand in this learning resource signifies that you as a learner is capable and empowered to successfully achieve the relevant competencies and skills at your own pace and time. Your academic success lies in your own hands!

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

What I Need to Know

This module was designed and written with you in mind. It is here to help you master the Inverse of a One-to-one Function. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

The module is outlined in this manner:

- Lesson 1: Representing real-life situations using one-to-one functions
- Lesson 2: Inverse of a one-to-one function.
- Lesson 3: Representing an inverse function through its:
  (a) table of values, and
  (b) graph.
- Lesson 4: Domain and Range of an inverse function.
- Lesson 5: Problems involving inverse functions.

After going through this module, you are expected to:

1. represent real-life situations using one-to one functions (M11GM-Id-1);
2. determine the inverse of a one-to-one function (M11GM-Id-2);
3. illustrate the inverse of a function using table of values and graph (M11GM-Id-3);
4. determine the domain and range of inverse functions (M11GM-Id-4); and
5. solve problems involving inverse functions (M11GM-Ie-2).

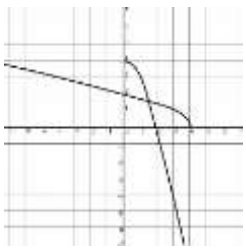
What I Know

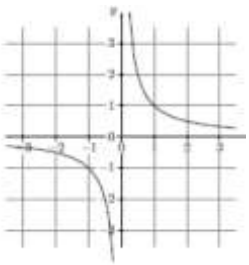
Multiple Choice. Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

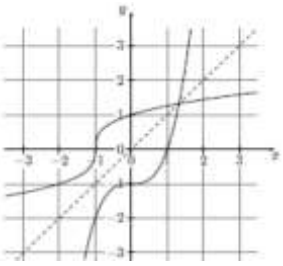
1. Which of the following ordered pair represents one-to-one functions?
   A. (0,2)(1,2)(2,2)
   B. (-2,0)(-2,1)(-2,-2)
   C. (0,1)(1,2)(2,3)
   D. (0,0)(1,3)(2,6)
2. One-to-one functions crosses a horizontal line _____ times.
   A. 0
   B. 1
   C. 2
   D. infinitely many
3. All of the following are one-to-one functions, EXCEPT:
   A. $f(x) = x$
   B. $f(x) = 1$
   C. $f(x) = x - 1$
   D. $f(x) = x + 1$
4. Complete the statement: A function is one to one if:
   A. exactly one domain corresponds to exactly one range.
   B. there is two domains in one range.
   C. in every domain there corresponds two ranges.
   D. many domain and many range.
5. Complete the statement: (-1,2) (1,2) (2,2) is:
   A. one to one
   B. one to many
   C. many to one
   D. many to many
6. The inverse of an inverse $f^{-1}(x)$ is _____.
   A. x
   B. 1
   C. $f^{-1}(x)$
   D. $f(x)$
7. $f^{-1}(f(x))$ = _____ for all x in the domain of f.
   A. y
   B. x
   C. 1
   D. 0
8. A function with an inverse is described to be _____.
   A. one-to-many
   B. one-to-one
   C. many-to-one
   D. many-to-many
9. Complete the statement: The inverse of a one-to-one function can be interpreted as the same function _____, that is, it is a function from a y-value back to its corresponding x-value.

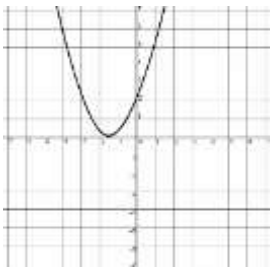
A. but in the same direction
B. but in the same value
C. but in the opposite direction
D. but in the same value

10. What is the result if a function that is not one-to-one is inverted?
    A. not a function
    B. a function
    C. a relation
    D. not a relation
11. What is the implication of a function satisfying the horizontal line test?
    A. It indicates that the function is linear.
    B. It indicates that the given is a one-to-one function.
    C. It indicates that the function does not satisfy the vertical line test.
    D. It indicates that different x values of the function can have exactly the same corresponding y values.
12. Which of the following are one-to-one functions?
    A. Books to authors
    B. People to family names.
    C. SIM cards to cell phone numbers
    D. True or False questions to answers
13. Which of the following is the inverse of the function $f(x) = 2x - 5$?
    A. $f^{-1}(x) = \frac{2}{x+5}$
    B. $f^{-1}(x) = \frac{5}{x+2}$
    C. $f^{-1}(x) = \frac{x+5}{2}$
    D. $f^{-1}(x) = \frac{x}{x+2}$
14. Which among the statements is NOT true about the horizontal line test?
    A. It is exactly the same with the vertical line test.
    B. It determines if the function tested is one-to-one.
    C. It is a verification that a function is symmetric with its inverse with respect to $y = x$.
    D. Graphs of a function and its inverse meet at utmost one point along the symmetry line.
15. Which graph below is NOT a one-to-one function?

A. 

B. 

C. 

D. 

16. Given a one-to-one function f(x) and its inverse f^(-1)(x). Then the following are true except one:
    A. The inverse of $f^{-1}(x)$ is $f(x)$
    B. $f(f^{-1}(x)) = x$ for all x in the domain of f^{-1}

C. $f^{-1}(f(x)) = x$ for all x in the range of f^{-1}
D. $f^{-1}(f(x)) = x$ for all x in the domain of f

17. What is the result if a function that is not one-to-one is inverted?
A. not a function B. a function C. a relation D. not a relation

18. Find the domain of this set of data. { (1,2), (3,4), (5,6), (7,8) }
A. { 2, 4, 6, 8} C. { 1, 2, 3, 4 }
B. { 1, 3, 5, 7 } D. { 5, 6, 7, 8 }

19. The range f of an inverse function is the _____ of the original function.
A. domain B. range C. Inverse D. Function

20. A function with an inverse is described to be _____.
A. one-to-many B. one-to-one C. many-to-one D. many-to-many

Lesson 1 Representing Real-life Situations Using One-to-one Functions

Have you ever asked yourself how your height and weight predict your BMI (Body Mass Index)? How does the power consumption affects the electric bill? How does the number of studying hours affect your grade? This module will discuss further how to determine one to one functions and its application in real life situations.

What's In

Group the following objects in such a way that the way that they common properties/characteristics.

triangle	guava	circle
pineapple	crayons	eraser
apple	rectangle	durian
pencil	book	yellow pad
oval	banana	square

Shapes	**Fruits**	**School Supplies**

Form some ordered pairs using the format:
a. Column 1 _____

b. Column 2 _____
c. Column 3 _____

Questions:

1. How many objects can be found in each column?
2. How did you classify the objects?
3. Based on the coordinates you have formulated, is there a repetition of the first coordinates? What about the second coordinates?

What's New

Representing Relations!

Given a table, you will be able to learn how to make a set of ordered pairs.

FOODS (x)	DRINKS (y)
Camote Que	Calamansi Juice
Banana Que	Pineapple Juice
Hotcake	Mango Juice
Pandesal	Apple Juice
Cheesebread	Orange Juice

1. What are the set of ordered pairs?
2. What elements belongs to the first set? Second set?
3. Is there a repetition of the first coordinates? How about the second coordinates?
4. Does the ordered pairs represent a relation?
5. How is the relation represented?

What is It

Represents Real Life Situations using One to One Functions

The previous discussion shows that

- Rational function is a function that is a fraction and has the property that both its numerator and denominator are polynomials. In other words, R(x) is a rational function if $R(x) = \frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are both polynomials, and $q(x) \neq 0$.

Definition

The function f is one to one if for any x_1 , x_2 in the domain of f then $f(x_1) \neq f(x_2)$. That is the same y-value is never paired with two different x values.

Points to Remember:

A relation can be presented in many ways:

A. Set of Ordered Pairs
B. Mapping Diagram
C. Table of Values
D. Graph
E. Rule/ Function

A relation has four types:

- One to one
- Many to one
- One to many
- Many to many; thus only one to one relation and many to one relation represents a function.

Example 1: Suppose the Kidzoona charge in an hour play is Php 300. Your baby brother wants to play in the said amusement.

Questions:

1. How much will you pay if your baby brother will play 1 hour? 2 hours? 3 hours? How about 5 hours?
2. Based on your answer in item 1, write an ordered pair in the form (time, amount).
3. Make a rule/ function that represents the given relation.
4. Is the given relation a one to one function?
5. How are time consume in Kidzoona related to charge to be paid?

Solutions:

1. If the brother will play 1 hour the sister will pay Php 300, while if he will play 2 hours his sister will pay Php 600, moreover if he will play for 3 hours they will pay Php 900.
2. Ordered pairs are: (1,300) (2,600) (3,900).
3. $f(x) = 300x$, thus x represents the time spent in Kidzoona while f(x) represents the total charge to be paid.
4. The relation is a one to one function since there exactly one element of f(x) in every element of x.
5. The total amount to be paid depends on the time consumed in Kidzoona.

Example 2: Suppose you want to call your father by telephone. The charge of a pay phone call is Php 10 pesos for the first 2 minutes and an additional Php 3 for every additional minute or a half of it.

Questions:

1. How much will you pay if you call your father for 1 minute? 2 minutes? How about 4 minutes?
2. Represents table of value of the given relation based on your answer in item 1.
3. Is the relation a one to one function? Why or why not?

Solutions:

1. If I will call my father for 1 minute I will pay Php 10, if 2 minutes Php 10 and if 4 minutes Php 16.
2.

Number of minutes of phone call (x)	1	2	4
Total amount (f(x))	Php 10	Php 10	Php 16

3. The relation is not a one to one function since there is only one element of f(x) in every element of x. Thus, it represents a many to one function.

Lesson 2 Inverse of a One-to-one Function

A function and its inverse can be described as the "DO" and the "UNDO" functions. A function takes a starting value, performs some operation on this value, and creates an output answer. The inverse function takes the output answer, performs some operation on it, and arrives back at the original function's starting value.

Inverting Functions

The previous discussion shows that

- if the x- and y-values of a one-to-one function are interchanged, the result is a function, but
- if the x- and y-values of a function that is not one-to-one are inverted, the result is no longer a function.

Definition

Let f be a one-to-one function with domain A and range B. Then the *inverse of* f, denoted by f^{-1}, is a function with domain B and range A defined by $f^{-1}(y) = x$ if and only if $f(x) = y$ for any y in B.

A function has an inverse if and only if it is one-to-one.

As shown earlier, 'inverting' the x- and y-values of a function results in a function if and only if the original function is one-to-one.

To find the inverse of a one-to-one function,

(a) write the function in the form $y = f(x)$;
(b) interchange the x and y variables;
(c) solve for y in terms of x. This is because we are interchanging the input and output values of a function.

Example 1: Find the inverse of $f(x) = 3x + 1$.

Solution:

a. write the function in the form $y = f(x)$. $\rightarrow \quad y = 3x + 1$
b. interchange the x and y variables. $\rightarrow \quad x = 3y + 1$
c. solve for y in terms of x. $\rightarrow \quad x = 3y + 1$

$$x - 1 = 3y$$

$$\frac{x-1}{3} = \frac{3y}{3}$$

$$\frac{x-1}{3} = y$$

Therefore, the inverse of $f(x) = 3x + 1$ is $f^{-1}(x) = \frac{x-1}{3}$.

Questions:

(a) What is the inverse of the inverse?

(b) What is $f(f^{-1}(x))$? How about $f^{-1}(f(x))$?

To answer these questions, use Example 1 before proceeding to the next part. Solve using a scratch paper and record the properties you observed.

- To solve for the inverse of $f^{-1}(x) = \frac{x-1}{3}$, we have $f^{-1}\left(f^{-1}(x)\right)$.

Solution:

$$y = \frac{x-1}{3}$$

$$x = \frac{y-1}{3}$$

$$(3)(x) = (\frac{y-1}{3})(3)$$

$$3x = y - 1$$

$$3x + 1 = y$$

$$3x + 1 = f(x)$$

Therefore, the inverse of $f^{-1}(x) = \frac{x-1}{3}$ is $f^{-1}\left(f^{-1}(x)\right) = f(x) = 3x + 1$.

- Solving for $f(f^{-1}(x))$ and $f^{-1}\left(f(x)\right)$, given the functions $f(x) = 3x + 1$ and $f^{-1}(x) = \frac{x-1}{3}$.

Solutions:

$$f\left(f^{-1}(x)\right) = 3\left(\frac{x-1}{3}\right) + 1$$

$$f\left(f^{-1}(x)\right) = x - 1 + 1$$

$$\boldsymbol{f\left(f^{-1}(x)\right) = x}$$

$$f^{-1}\left(f(x)\right) = \frac{(3x+1)-1}{3}$$

$$f^{-1}\left(f(x)\right) = \frac{3x}{3}$$

$$\boldsymbol{f^{-1}\left(f(x)\right) = x}$$

From the above solutions, we can summarize the properties of an inverse of a one-to-one function.

Property of an inverse of a one-to-one function

Given a one-to-one function $f(x)$ and its inverse $f^{-1}(x)$. Then the following are true:

(a) The inverse of $f^{-1}(x)$ is $f(x)$.
(b) $f\left(f^{-1}(x)\right) = x$ for all x in the domain of f^{-1}.
(c) $f^{-1}\left(f(x)\right) = x$ for all x in the domain of f.

Example 2: Find the inverse of $g(x) = x^3 - 2$.

Solution:

$$y = x^3 - 2$$
$$x = y^3 - 2$$
$$x + 2 = y^3$$
$$\sqrt[3]{x+2} = \sqrt[3]{y^3}$$
$$\sqrt[3]{x+2} = y$$
$$\sqrt[3]{x+2} = g^{-1}(x)$$

Therefore, the inverse of $g(x) = x^3 - 2$ is $g^{-1}(x) = \sqrt[3]{x+2}$.

Example 3: Find the inverse of $f(x) = x^2 + 4x - 2$ if it exists.

Solution:

$$y = x^2 + 4x - 2$$

$$x = y^2 + 4y - 2$$
$$x + 2 = y^2 + 4y$$
$$x + 2 + 4 = y^2 + 4y + 4 \quad \textit{(Complete the square)}$$
$$x + 6 = (y + 2)^2$$
$$\sqrt{x + 6} = \sqrt{(y + 2)^2}$$
$$\sqrt{x + 6} = y + 2$$
$$\pm\sqrt{x + 6} - 2 = y$$
$$f^{-1}(x) = \pm\sqrt{x + 6} - 2$$

The equation $\pm\sqrt{x + 6} - 2 = y$ **does not** represent a function because there are some x-values that correspond to two different y-values (e.g., if $x = 3$, y can be 1 or -5).
Therefore, the function $f(x) = x^2 + 4x - 2$ has **no inverse function.**

Example 4: Find the inverse of $f(x) = |4x|$, if it exists.

Solution: Recall that the graph of $y = |3x|$ is shaped like a "V" whose vertex is located at the origin. This function fails the horizontal line test and therefore has no inverse.

Alternate Solution: We can also show that f^{-1} does not exist by showing that f is not one-to-one. Note that $f(1) = f(-1) = 4$. Since the x-values 1 and -1 are paired to the same y-value, then f is not one-to-one and it cannot have an inverse.

If we apply the procedure in solving the inverse of a one-to-one function:

$$y = |4x|$$
$$x = |4y|$$
$$x = \sqrt{(4y)^2} \quad \textit{(Recall the definition } |x| = \sqrt{x^2}\textit{)}$$
$$x^2 = 4y^2$$
$$\frac{x^2}{4} = y^2$$
$$\pm\sqrt{\frac{x^2}{4}} = y$$

Here, $x = 2$ will correspond to $y = 1$ and $y = -1$, so $y = \pm\sqrt{\frac{x^2}{4}}$ is **not** a function. Therefore, $f(x) = |4x|$ has **no inverse function**.

Lesson 3 Table and Graph of an Inverse Function

If function passes the horizontal line test, it is an indication that the said function is one-to-one. Subsequently, its inverse also is a function.

Since the domain of the function becomes the range of its inverse, and the range of the function becomes the domain of its inverse, in problems dealing with a function and its inverse, when determining the inverse' ordered pairs, we simply reverse the domain and range of the original function.

Example 1

Find the inverse of the function $f(x) = 2x + 1$; present the graph of the original function and its inverse.

$f(x) = 2x + 1$

x	-1	0	1	2
y				

In order to accomplish a table of values such this, first, you must set values of your *x.*

Then, to obtain corresponding values of *y,* substitute each value of *x* in the function.

We have the first value of *x* that is -1,

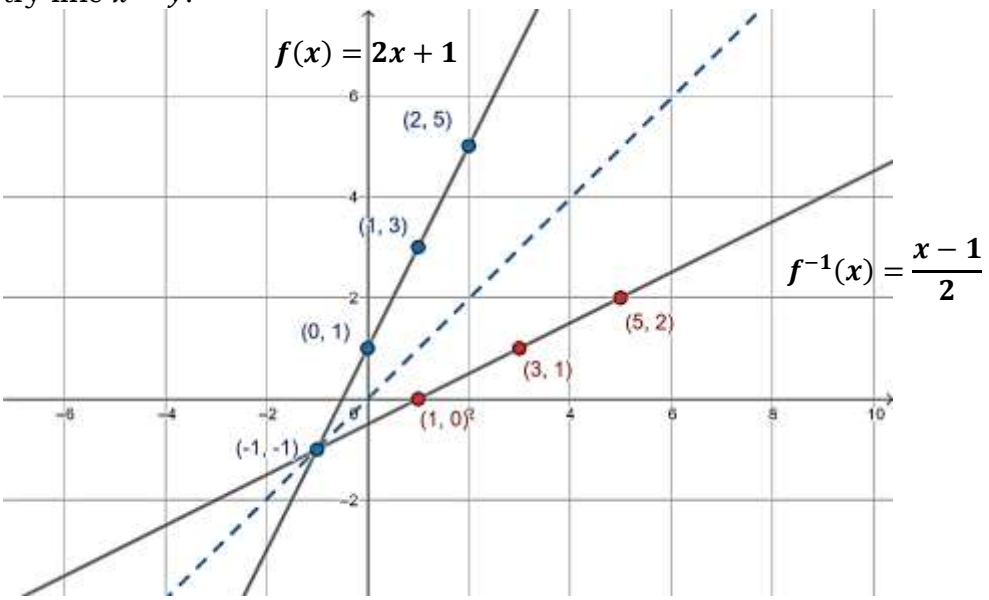
$y=2x+1$
$y=2(-1)+1$
$y=-2+1$
$y=-1.$

The accomplished table of ordered pairs of the original function $f(x) = 2x + 1$ and its inverse is presented below.

$f(x) = 2x + 1$ $\qquad$ $f^{-1}(x) = \frac{x-1}{2}$

x	-1	0	1	2
y	-1	1	3	5
x	-1	1	3	5
y	-1	0	1	2

The graphs of the given function and its inverse are reflections of each other along the symmetry line $x = y$.

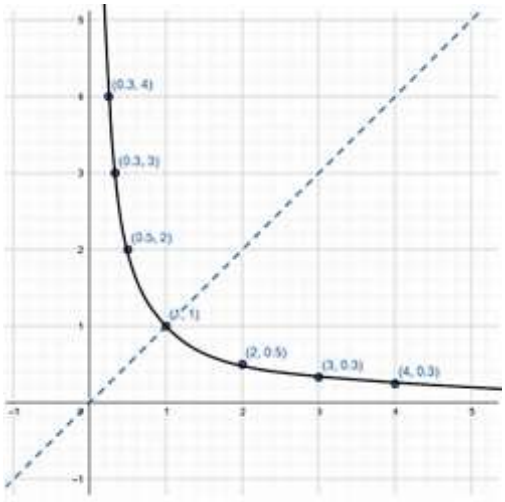


Example 2

Here we have a rational function. Observe its representation in table and graphs.

$f(x) = \frac{1}{x}$ $\qquad$ $f^{-1}(x) = \frac{1}{x}$

x	1	2	3	4
y	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$
x	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$
y	1	2	3	4



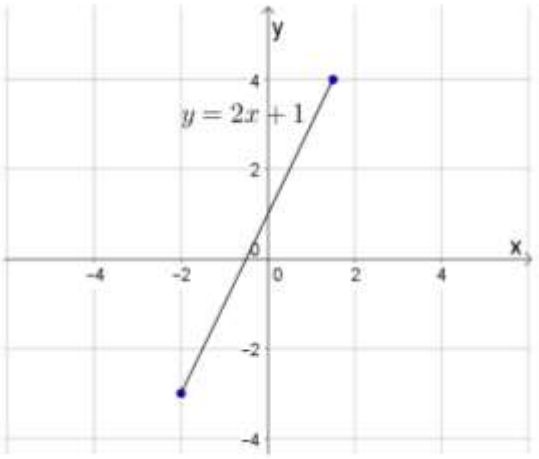
It is notable that the inverse function is the same with the original function. In this case, its reflection across the line $y = x$ is itself.

Lesson 4 Domain and Range of an Inverse Function

Since the domain of the function becomes the range of its inverse, and the range of the function becomes the domain of its inverse, in problems dealing with a function and its inverse, when determining the inverse' ordered pairs, we simply reverse the domain and range of the original function.

EXAMPLE 1.

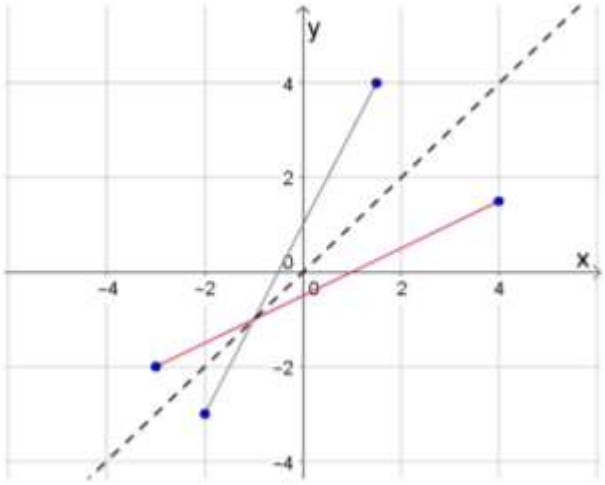
Graph $y = f^{-1}(x)$ if the graph of $y = f(x) = 2x + 1$ restricted in the domain $\{x \mid -2 \leq x \leq 1.5\}$ is given below. What is the range of $f(x)$? What is the domain and range of its inverse?



Solution.

Take the reflection of the restricted graph of $y = 2x + 1$ across the line $y = x$.

The range of the original function can be determined by inspection of the graph. The range is $\{y \in \mathrm{R} \mid -3 \leq y \leq 4\}$.
Domain $[-2, 1.5]$ $[-3, 4]$
Range $[-3, 4]$ $[-2, 1.5]$



In summary:

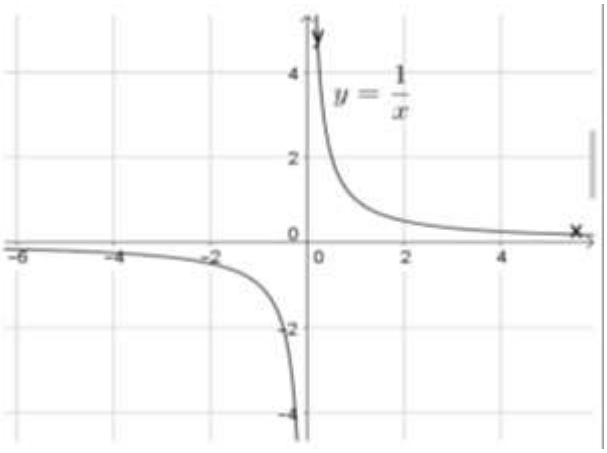
	$f(x)$	$f^{-1}(x)$
Domain	[-2, 1, 5]	[-3, 4]
Range	[-3, 4]	[-2, 1, 5]

Observe that the domain of the inverse is the range of the original function, and that the range of the inverse is the domain of the original function. Is this true for all one-to-one functions and their inverses?

EXAMPLE 2.
Find the domain and range the inverse of $f(x) = \frac{1}{x}$ whose graph is shown below.

Solution.

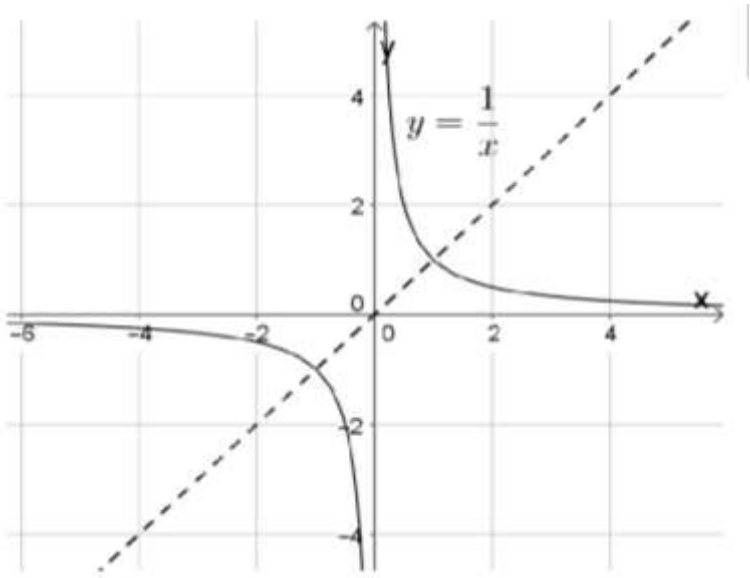
Applying the horizontal line test we verify that the function is one-to-one. Since the graph of $f(x) = \frac{1}{x}$ is symmetric with respect to the line $y = x$ (indicated by a dashed line), its reflection across the line $y = x$ is itself. Therefore the inverse of $f(x)$ is itself, or $f^{-1}(x) = f(x)$.



In summary:

The domain and range of the inverse function can be determined by inspection of the graph:

Domain of $f^{-1}(x) = [-3, 4]$
Range of $f^{-1}(x) = [-2, 1.5]$



Lesson 5 Problems Involving Inverse Functions

Inverse function is something that tells you how to get back to its original phase, value, form or its reverse, and problems involving inverse functions applies a lot in real life scenario. So basically it, requires the skills of formulating the mathematical expression out from mathematical statement in deriving the correct answer.

Remember in solving a problem.

1. Understand the problem.
   The problem ask if the temperatures in Philippines is equivalent to the temperature in Fahrenheit in UK during the conversation.
2. What are the clues given?
   You said that its 350C is the temperature in the Philippines. Her friend said that its 920F in UK at that moment, and concluded that the two are the equivalent in reverse.
3. What is the formula to convert Celsius to Fahrenheit?
   To convert Celsius to Fahrenheit, use the formula:
   $$F = 1.8\,c + 32$$
4. What is the formula to convert Fahrenheit to Celsius?
   The formula is:
   $$C = (F - 32)/1.8$$
5. Carry out the plan. By using the formula substitute the given.
6. Check and review your answer.

Solution:

Step 1. Write the mathematical model in converting Celsius to Fahrenheit. This is your original function.

$$F = 1.8\,c + 32$$

Step 2. Keep in mind the steps in solving an inverse function.

In this problem your y (K) and X (c).

Step 1. Change K to Y that is: $y = 1.8\,x + 32$
Step 2. Interchange y and x: $x = 1.8\,y + 32$
Step 3. Solve for new y in terms of x:
Apply Subtraction Property of Equality $x - 32 = 1.8\,y + 32$
$x - 32 = 1.8\,y - 32$
$x - 32 = 1.8\,y$

To eliminate 1.8 divide both sides by it:

$$\frac{x-32}{1.8} = \frac{1.8y}{1.8}$$

What's left will be:

$$\frac{x-32}{1.8} = y$$

Step 4. The new equation or the f^{-1} is:

$$y = \frac{x-32}{1.8}$$

Therefore,
Step 3. The inverse of
$F = 1.8\,c + 32$ is $C = (F - 32)/1.8$
Let's check if 35 o C is an inverse of 95^0 F.

$F = 1.8\,c + 32$
$F = 1.8\,(35) + 32$
$F = 63 + 32$
$F = 95$

The problem in previous activity tackles about inverse function since Celsius and Fahrenheit are temperature set up. With Fahrenheit you're really get cold at 0°F and really hot at 100°F and with Celsius you're cold at 0°C and dead at 100°C.

From the given data above we can tell, that the two are inversely equivalent. Expressing a mathematical statement into mathematical expression / model is a necessary skill to solve problems involving inverse function.

What's More

Activity 1. IDENTIFY THE RELATION
Direction: Determine the types of relation being described. Choose your answer from **one to one**, **many to one**, **one to many** or **many to many**.

1. (2,3) (3,4) (4,5) 2. (0,1) (0,2) (0,3) 3. (-1,-1) (-2,-1) (-3,-1)

4.

x	2	2	2
f(x)	0	-1	-2

5.

x	5	10	15
f(x)	10	20	30

Activity 2. COMPLETING THE SOLUTION

In this activity you will be required to fill-in the missing steps in the solution for finding the inverse of a one-to-one function. You will select your answers from the box given at the right side. Good luck!

1. Solve for the inverse of $f(x) = 4x - 1$.

Solution:

$y = 4x - 1$

(a) ☐

$x + 1 = 4y$

$\frac{x+1}{4} = y$

(b) ☐

$x = 4y - 1$
$y = 4x + 1$
$\frac{y+1}{4} = x$
$f^{-1}(x) = \frac{x+1}{4}$
$x - 1 = 4y$

2. Complete the solution for the inverse of the function $h(x) = \frac{2x+1}{5}$ by arranging all the steps found inside the box.

Solution:

(i) ☐

(ii) ☐

(iii) ☐

(iv) ☐

(v) ☐

$h^{-1}(x) = \frac{5x-1}{2}$

(a)	$5x = 2y + 1$
(b)	$y = \frac{2x+1}{5}$
(c)	$5x - 1 = 2y$
(d)	$\frac{5x-1}{2} = y$
(e)	$x = \frac{2y+1}{5}$

Activity 3. COMPLETING THE TABLES

Let us practice completing table of values of an original function and its inverse.

Function

1. $f(x) = 2x$

x	-1	0	1	2
y	-2		2	4

Inverse

$f^{-1}(x) = \frac{x}{2}$

x		0	2	
y				

2. $f(x) = 3x - 5$ $\quad f^{-1}(x) = \frac{x-5}{3}$

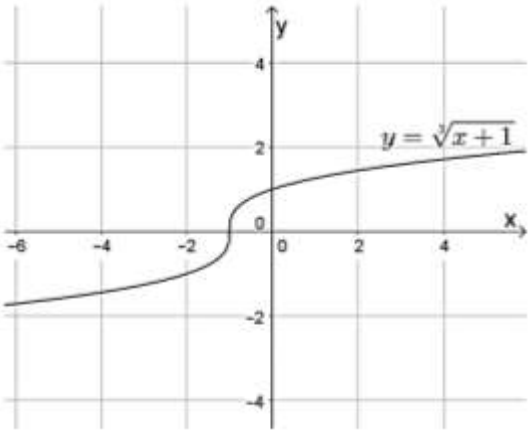
x	0	1	2	3
y	-5			
x	-5			
y				

3. $f(x) = x^3$ $\quad f^{-1}(x) = \sqrt[3]{x}$

x	-2	-1	0	1	2	3
y						
x						
y						

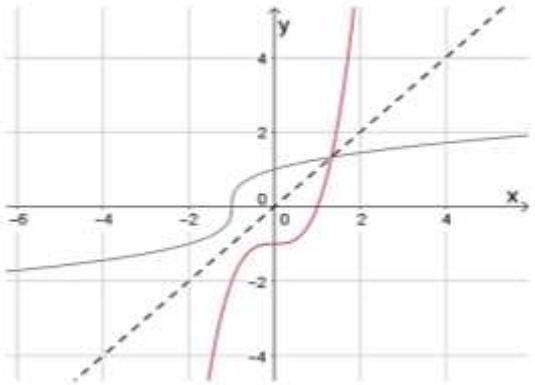
Activity 4. FINDING THE DOMAIN AND RANGE

1. Find the inverse of $f(x) = \sqrt[3]{x+1}$ using the given graph.
The domain and range of the inverse function can be determined by inspection of the graph.



Solution:

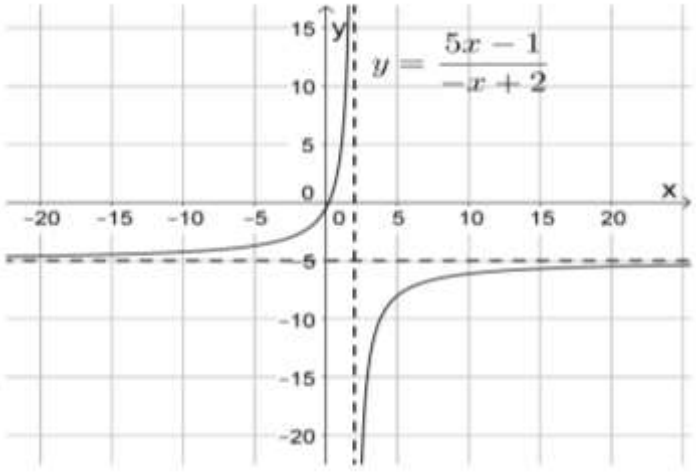
Applying the horizontal line test we confirm that the function is one-to-one.



Reflect the graph of $f(x)$ across the line $y = x$ to get the plot of the inverse function. The result of the reflection of the graph of $f(x) = \sqrt[3]{x+1}$ is the graph of $y = x^3 - 1$.

Therefore $f^{-1}(x) = x^3 - 1$.

2. Consider the rational function $f(x) = \frac{5X-1}{-X+2}$ whose graph is shown below.



(a) Find its domain and range.
(b) Find the equation of its asymptotes.
(c) Find the graph of its inverse.
(d) Find the domain and range of its inverse.

Solution.

(a) From our lessons on rational functions, we get the following results:

Domain of $f(x) = (-\infty, 2) \cup (2, \infty)$

Range of $f(x) = (-\infty, -5) \cup (-5, \infty)$

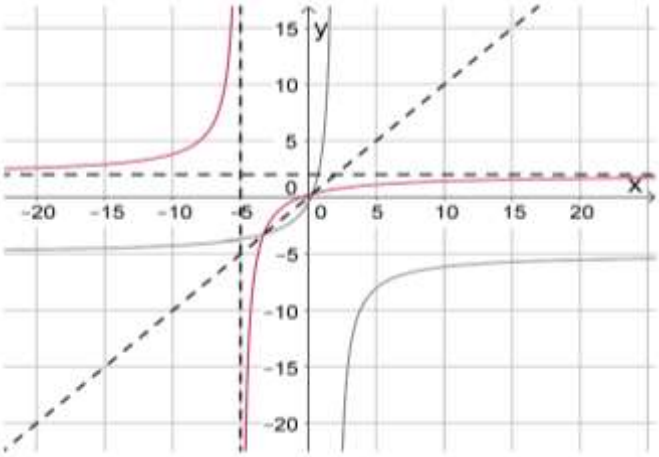
(b) Using techniques from the lesson on rational functions, the equations of the asymptotes are

Vertical asymptote: $x = 2$

Horizontal asymptote: $y = -5$

(c) The inverse can be graphed by taking the reflection of the graph across $y = x$.

Observe that the new asymptotes are the old asymptotes with the x and y values interchanged. In fact, the asymptotes could also be obtained by reflecting the original asymptotes about the line $y = x$.



What I Have Learned

A. Based on the concepts that you learn from this module, complete the following:

1-2. A relation is a function if it is a _____ and _____.

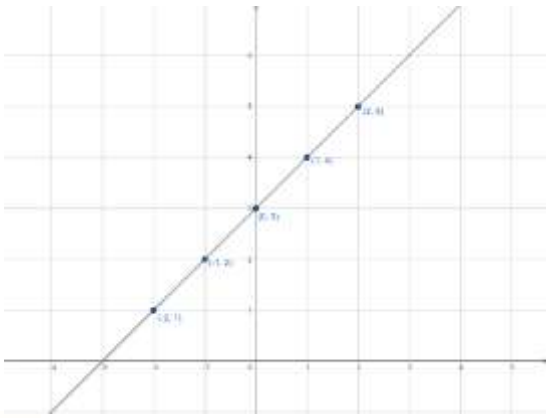
3. (2,3) (3,4) and (4,5) represents a _____ type of function. Given the table below.

Months of plants grown (x)	2	3	4
Height of the plants in cm (f(x))	21	31	41

4. Is the given a one to one function?

5. Make a function that represents the given situation.
6. Interchanging the x and y-values of a one-to-one function would result to a _____.
7. The inverse of f is denoted by _____.
8. A function has an inverse if and only if it is _____.
9. _____ for all x in the domain of f^{-1}
10. _____ for all x in the domain of f.

B. Below is the graph of the function $f(x) = x + 3$. Find its inverse, complete its set of values, and represent it with a graph. (Construct the graph in the same plane below.)



$f^{-1}(x) =$

x	1	2	3	4	5
y					

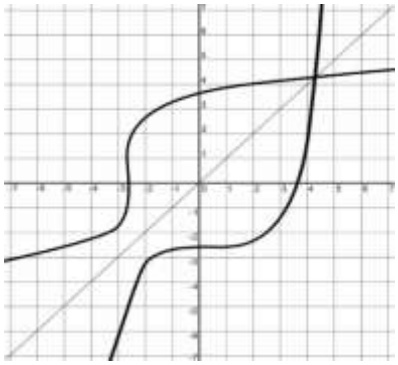
What I Can Do

This section involves real-life application of the inverse of a one-to-one function. Read and understand the problem carefully and show your complete solution.

A. Theresa's mother asked her to buy dressed chicken. If a kilo of dressed chicken costs Php 180, how many kilos she can buy if her mother gave her Php 450? Is the given situation represents a one to one function? If yes, generate a function that represents the situation.

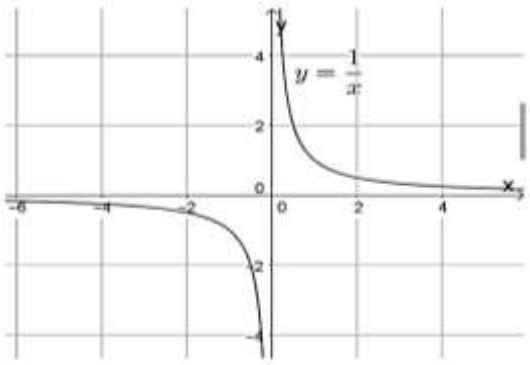
Solution:

B. Below is the graph of a function and its inverse. Is the original function one-to-one? Explain your thoughts.



_____
_____
_____
_____
_____
_____
_____

C. 1. Find the domain and range of the inverse function $f(x)=\frac{1}{x}$, whose graph is shown below.



2. Find the domain and range of the inverse function $f(x)=-x+1$

3. Find the domain and range of the function inverse $f(x)=\frac{3x+2}{x-4}$.

Assessment

Multiple Choice. Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. Which of the following ordered pair represents one to one functions.
   A. (0,2)(1,2)(2,2) C. (0,1)(1,2)(2,3)
   B. (-2,0)(-2,1)(-2,-2) D. (0,0)(1,3)(2,6)
2. One to one functions crosses a horizontal line _____ times.
   A. 0 B. 1 C. 2 D. infinitely many
3. All of the following are one to one functions, EXCEPT:
   A. $f(x)=x$ C. $f(x)=x-1$
   B. $f(x)=1$ D. $f(x)=x+1$
4. Complete the statement: A function is one to one if:
   A. exactly one domain corresponds to exactly one range
   B. there is two domains in one range.
   C. in every domain there corresponds two ranges.
   D. many domain and many range.
5. Complete the statement: (-1,2) (1,2) (2,2) is:
   A. one to one C. many to one
   B. one to many D. many to many
6. A function with an inverse is described to be _____.
   A. one-to-many C. many-to-one
   B. one-to-one D. many-to-many
7. The inverse of an inverse $f^{-1}(x)$ is _____.
   A. x
   B. 1
   C. $f^{-1}(x)$
   D. $f(x)$

8. What is the result if a function that is not one-to-one is inverted?
   A. not a function
   B. a function
   C. a relation
   D. not a relation
9. $f^{-1}(f(x)) =$ _____ for all x in the domain of f.
   A. y
   B. x
   C. 1
   D. 0
10. Complete the statement: The inverse of a one-to-one function can be interpreted as the same function_____, that is, it is a function from a y-value back to its corresponding x-value.
   A. but in the same direction
   B. but in the same value
   C. but in the opposite direction
   D. but in the same value

TRUE OR FALSE. Write TRUE if the statement is correct and FALSE if the statement is wrong.

_____11. A function has an inverse if it is one-to-many.
_____12. Given the graph of one-to-one function, the graph of its inverse can be obtained by reflecting the graph about the line.
_____13. This example is one-to-one? The relation pairing an SSS member to his or her SSS number.
_____14. The domain of a function is the set of all values that the variable can take.
_____15. The range of the function is the set of all values that will take.

Additional Activities

This section includes supplementary activities related to the inverse of a one-to-one function.

1. Graph $y = f^{-1}(x)$ if the graph of $y = f(x) = 2x + 5$ restricted in the domain $\{x|-2 \leq x \leq 1.5|\}$.
   a. What is the range of the function?
   b. What is the domain and range of its inverse?

2. Dan can paint the room in 3 hours, Jill can paint the same room in 5 hours. How long will it take to paint the room if they work together?

What I Know

1. C
2. B
3. B
4. A
5. B
6. D
7. B
8. B
9. C
10. A
11. B
12. C
13. C
14. A
15. D
16. C
17. C
18. B
19. A
20. A

Assessment

1. C
2. B
3. B
4. A
5. B
6. B
7. D
8. A
9. B
10. C
11. FALSE
12. TRUE
13. TRUE
14. FALSE
15. TRUE

What's More

Activity 1

1. one to one
2. one to many
3. many to one
4. one to many
5. one to one

Activity 2

1 a. $x = 4y - 1$

b. $f^{-1}(x) = \frac{x+1}{4}$

2 i. b
ii. e
iii. a
iv. c
v. d

Activity 3

$f(x) = 2x$

x	-1	0	1	2
y	-2	0	2	4

$f^{-1}(x) = \frac{x}{2}$

x	-2	0	2	4
y	-1	0	1	2

$f(x) = 3x - 5$

x	0	1	2	3
y	-5	-2	1	4

$f^{-1}(x) = \frac{x-5}{3}$

x	-5	-2	1	4
y	0	1	2	3

$f(x) = x^3$

x	-2	-1	0	1	2	3
y	-8	-1	0	1	8	27

$f^{-1}(x) = \sqrt[3]{x}$

x	-8	-1	0	1	8	27
y	-2	-1	0	1	2	3

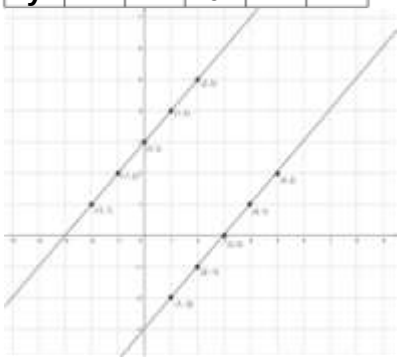
What I Have Learned

A.

1-2. one to one, many to one
3. one to one
4. yes
5. $f(x) = 10x + 1$
6. function
7. f^{-1}
8. one-to-one
9. $f\left(f^{-1}(x)\right) = x$
10. $f^{-1}\left(f(x)\right) = x$

B. $f^{-1}(x) = x - 3$

x	1	2	3	4	5
y	-2	-1	0	1	2



Answer Key

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DISCLAIMER

This Self-learning Module (SLM) was developed by DepEd SOCCSKSARGEN with the primary objective of preparing for and addressing the new normal. Contents of this module were based on DepEd's Most Essential Learning Competencies (MELC). This is a supplementary material to be used by all learners of Region XII in all public schools beginning SY 2020-2021. The process of LR development was observed in the production of this module. This is version 1.0. We highly encourage feedback, comments, and recommendations.

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