11

General Mathematics

Quarter 1 – Module 2:
Rational Functions

SELF-LEARNING MODULE

SLM

DEPARTMENT OF EDUCATION - SOCCSKSARGEN

General Mathematics – Grade 11
Self-Learning Module (SLM)
Quarter 1 – Module 2: Rational Functions
First Edition, 2020

Development Team of the Module

Writers: Arnel C. Lapitan; Richard O. Hemady; Venus Jane E. Rosete; Mary Jane N. Mizell; Hannih Lou T. Bantilan; Ana Cristy P. Mercado

Editors: Nathaniel A. Galopo

Reviewers: Nathaniel A. Galopo, Reynaldo C. Tagala

Illustrator:

Layout Artist: Maylene F. Grigana

Cover Art Designer: Ian Caesar E. Frondoza

Management Team: Allan G. Farnazo, CESO IV – Regional Director
Fiel Y. Almendra, CESO V – Assistant Regional Director
Gildo G. Mosqueda, CEO VI – Schools Division Superintendent
Diosdado F. Ablanido, CPA – Assistant Schools Division Superintendent
Gilbert B. Barrera – Chief, CLMD
Arturo D. Tingson Jr. – REPS, LRMS
Peter Van C. Ang-ug – REPS, ADM
Jade T. Palomar – REPS, Mathematics
Donna S. Panes – Chief, CID
Elizabeth G. Torres – EPS, LRMS
Judith B. Alba – EPS, ADM
Reynaldo C. Tagala – EPS, Mathematics

Printed in the Philippines by Department of Education – SOCCSKSARGEN Region

Office Address: Regional Center, Brgy. Carpenter Hill, City of Koronadal
Telefax: (083) 2288825/ (083) 2281893
E-mail Address: region12@deped.gov.ph

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-test are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module, or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teachers are also provided to the facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. Read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.
Thank you.

What I Need to Know

This module was designed and written with you in mind. It is here to help you master the key concepts on Rational Functions. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

This module presents lesson in this outline:

1. Definition of Rational Functions
2. Comparison of Rational Function, Rational Equation and Rational Inequality;
3. Solving Rational Equation and Inequality
4. Representing of Rational Function through Table of Values, Graph, and Equation
5. Domain and Range of rational Function

After going through this module, you are expected to:

1. define a rational function;
2. identify situations that model rational function;
3. write an equation representing situations that model rational function ;
4. distinguish rational function, rational equation and rational inequality;
5. solve rational equations and inequalities;
6. represent rational functions through table of values, graph and equation; and
7. find the domain and range of rational function

What I Know

Multiple Choice: Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. Which of the following is an example of rational function?

   A. $f(x) = \frac{1}{x}$ C. $f(x) = \frac{x^2}{3x+4}$

   B. $f(x) = \frac{4}{4-3x}$ D. All of the above

2. A motorcycle travels a distance of 80 meters. Express velocity **v** as a function of travel time **t**, in seconds.

   A. $v = \frac{80}{t}$ C. $v = 80t$

   B. $v = \frac{t}{80}$ D. $v = 80^t$

3. Which of the following relationships of physical quantities can be modeled by rational function?

   A. The circumference of a circle related to its radius.

   B. The child's dose related to his age taking adult's dose to be constant.

   C. The voltage form a source related related the current flowing in a wire.

   D. The weight related to amount of food intake.

Fot items 4 to 6, identify whether the given mathematical statement is a Rational Function, Rational Equation, Rational Inequality or None of these. Choices are provided nside the box. Write the chosen letter on a separate sheet of paper.

A. Rational Function	C. Rational Inequality
B. Rational Equation	D. None of these

4. $\frac{7}{x-1} \leq \frac{2}{x+2}$

5. $\frac{4x}{6} = \frac{5x+2}{8x^2+x}$

6. $f(x) = \sqrt{x+1}$

7. Which of the following is a solution of the equation $x - \frac{18}{x} = 3$?

A. 3
B. 4
C. 5
D. 6

8. Which of the following is NOT a solution to the inequality $\frac{2}{x+3} > \frac{x}{2}$?

A. -3
B. -2
C. -1
D. 0

9. What is the solution set of the inequality $\frac{5x}{x-1} < 4$?

A. $\{x|x \in R, -4 < x < 1\}$
B. $\{x|x \in R, x < 1\}$
C. $\{x|x \in R, x > 1\}$
D. $\{x|x \in R, x \neq 1\}$

10. Which rational function is represented by the table of values below?

x	1	2	3	-1	-2
$f(x)$	$\frac{4}{3}$	$\frac{8}{5}$	$\frac{12}{7}$	4	$\frac{8}{3}$

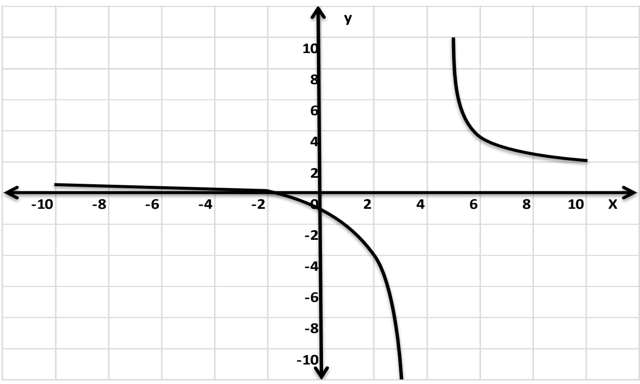
A. $f(x) = \frac{2x}{4+x}$

C. $f(x) = \frac{4x}{2x+1}$

B. $f(x) = \frac{2x}{2+x}$

D. $f(x) = \frac{2+x}{4+x}$

11. Which equation best represents the graph?



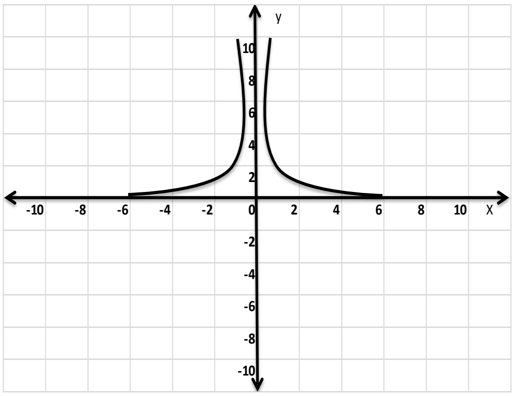
A. $f(x) = \frac{x-2}{x-4}$

C. $f(x) = \frac{x+2}{x+4}$

B. $f(x) = \frac{x+4}{x-2}$

D. C. $f(x) = \frac{x+2}{x-4}$

12. Which family of functions does $f(x)=\frac{4}{x^2}$ belong to?



A. Trigonometric

B. Exponential

C. Logarithmic

D. Rational

13. What value of x in $f(x)=\frac{5-x}{x+3}$ will make the function undefined?

A. $-\frac{3}{5}$

B. -3

C. 3

D. 5

14. The domain of the function $f(x)=\frac{x}{x+11}$ is the set of all real numbers except ________.

A. -11

B. 1

C. 0

D. 11

15. What is the range of the functon in item 14?

A. all real numbers except -11

B. all real numbers except 1

C. all real numbers except 0

D. all real numbers except 11

Lesson 1 Rational Functions

Rational function is just one of the many functions that can model relationships of physical quantities or variables dealt in different fields. Some real-world applications of rational function are electronic circuitry and optics (physics), spectroscopy (chemistry), concentration of drugs (medicine) and a lot more. In this module, discussion of key concepts of rational functions is of concern.

What's In

In Grade 8 mathematics, you learned about **rational algebraic expression**, an expression of the form $\frac{p(x)}{q(x)}$ where p(x) and q(x) are both polynomials. As a sort of review about rational algebraic expression, do the following activity.

Given a certain value of *x*, determine the value of the rational algebraic expression:

1. $\frac{x}{x-2}$; $x=-2$.

2. $\frac{x+1}{x}$; $x=1$.

3. $\frac{2x}{3}$; $x=6$.

4. $\frac{3}{x^2-x+1}$; $x=-1$.

5. $\frac{x-4}{x^2+6x-5}$; $x=2$.

What's New

There are a lot of situations in real-life that are modelled by rational functions. To help you understand the concept about rational function, consider this situation and then do the accompanying activity.

The local barangay received a budget of P100,000 to provide medical checkups for the children in the barangay. The amount is to be alloted equally among the children in the barangay.

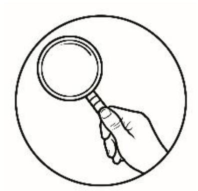
1. Fill-up the table below with the different allotment amounts for different values for the number of children:

No. of Children (x)	10	20	50	100	200	500	1000
Alloted Amount (y)							

2. Write an equation representing the relationship of the alloted amount per child (y-variable) versus the total number of children (x-variable).

Notes to the Teacher

Writing an equation of a relationship between variables x and y is the same as expressing $y = f(x)$, where y is a function of x.



What is It

1.1 Definition of Rational Function

A **rational function** is a function of the form $f(x) = \frac{p(x)}{q(x)}$, where p(x) and g(x) are polynomial function and g(x) is not the zero function (i.e. $q(x) \neq 0$). The domain of f(x) is the set of all values of x where $q(x) \neq 0$.

Example 1. A Car is to travel a distance of 20 meters. Express velocity **v** as a function of travel time **t**, in seconds.

Solution: The following table of values show ***v*** for various values of ***t.***

t (seconds)	1	2	4	5	10
v (meters per second)	20	10	5	4	2

The function $v(t) = \frac{20}{t}$ can represent v as a function of t.

Example 2. A fence is to enclose a rectangular vegetable farm with an area of 400 square meters. If x is the length of one side of this fence, find a function $P(x)$ representing the perimeter of the fencing material required.

Solution: The following table of values show $P(x)$ for various values of ***x.***

x	2	10	20	50	100	200
$P(x)$	404	100	80	116	208	404

The function $P(x) = \frac{2x^2 + 800}{x}$ can represent v as a function of t.

1.2 Rational Function, Rational Equation and Rational Inequality

If fractions are studied in arithmetic and are frequently used in everyday life, rational expressions must equally be studied because of its usefulness in all fields to which algebra is applied. The comparison of rational equations, inequalities, and functions are shown below.

	Rational Equation	**Rational Inequality**	**Rational Function**
Definition	An equation involving rational expressions.	An Inequality involving rational expression	A function of the form $f(x)=\frac{p(x)}{q(x)}$ where p(x) and q(x) are polynomial functions and q(x) is not the zero function.

To help you distinguish the difference among the Rational Function, Rational Equation, and Rational Inequality, observe these examples:

1. $\frac{5x+2}{3}+\frac{x}{4}=15$ — Rational Equation
2. $2x+3<123$ — Rational Inequality
3. $\frac{a^2+6a+5}{a+1}=2a+4$ — Rational Equation
4. $f(x)=\frac{x^2-25}{x}$ — Rational Function
5. $6x+4\geq 8$ — Rational Inequality
6. $y=\frac{2}{x-4}$ — Rational Function
7. $\sqrt[3]{x}=x-5$ — None of these (because the expression involves radicals)

A rational equation or inequality can be solved for all x values that satisfy the equation or inequality. A rational function expresses a relationship between two variables (such as x and y), and can be represented by a table of values or a graph.

1.3 Solving Rational Rational Equation and Rational Inequality

To solve an equation or inequality in one variable as *x* means to find all values of *x* for which the equation or inequality is true.

Procedure for Solving Rational Equations:

(a) Eliminate denominators by multiplying each term of the equation by the least common denominator.
(b) Note that eliminating denominators may introduce extraneous solutions. Check the solutions of the transformed equations with the original equation.

Example 1. Solve for the value of x: $\frac{x}{5}+\frac{1}{4}=\frac{x}{2}$

Solution:

LCD: 20	Get the LCD of 5, 4 and 2.
$20\left(\frac{x}{5}+\frac{1}{4}\right)=20\left(\frac{x}{2}\right)$	Multiply both sides by the LCD, 20.
$4x+5=10x$	Apply the Distributive Property and then simplify.
$5=6x$	Subtract 4x from both sides.
$\frac{5}{6}=x$	Divide both sides by 6.

Check: $\frac{\frac{5}{6}}{5}+\frac{1}{4}=\frac{\frac{5}{6}}{2}$

$\left(\frac{5}{6}\right)\left(\frac{1}{5}\right)+\frac{1}{4}=\left(\frac{5}{6}\right)\left(\frac{1}{2}\right)$

$\frac{2+3}{12}\stackrel{?}{=}\frac{5}{12}$

$\frac{5}{12}\stackrel{\checkmark}{=}\frac{5}{12}$

Example 2. Solve for the value of x: $\frac{2}{x}-\frac{3}{2x}=\frac{1}{5}$

Solution:

LCD: $10x$	Get the LCD of x, $2x$ and 5.
$10x\left(\frac{2}{x}-\frac{3}{2x}\right)=10x\left(\frac{1}{5}\right)$	Multiply both sides by the LCD, 20.
$20-15=2x$	Apply the Distributive Property and then simplify.
$5=2x$	Subtract 4x from both sides.
$\frac{5}{2}=x$	Divide both sides by 6.

Check: $\frac{2}{\frac{5}{2}}-\frac{3}{2\left(\frac{5}{2}\right)}=\frac{1}{5}$

$\left(\frac{2}{1}\right)\left(\frac{2}{5}\right)-\left(\frac{3}{1}\right)\left(\frac{1}{5}\right)=\frac{1}{5}$

$\frac{4}{5}-\frac{3}{5}\stackrel{?}{=}\frac{1}{5}$

$\frac{1}{5}\stackrel{\checkmark}{=}\frac{1}{5}$

So, $\frac{5}{2}$ is the solution.

Procedure for Solving Rational Inequalities:

(a) Rewrite the inequality as a single rational expression on one side of the inequality symbol and 0 on the other side.

(b) Determine over what intervals the rational expression takes on positive and negative values.

i. Locate the x values for which the rational expression is zero or undefined (factoring the numerator and denominator is a useful strategy).

ii. Mark the numbers found in (i) on a number line. Use a shaded circle to indicate that the value is included in the solution set, and a hollow circle to indicate that the value is excluded. These numbers partition the number line into intervals.

iii. Select a test point within the interior of each interval in (ii). The sign of the rational expression at this point is also the sign of the rational expression at each interior point in the aforementioned interval.

iv. Summarize the intervals containing the solutions.

Interval and Set Notation

An inequality may have infinitely many solutions. The set of all solutions can be expressed using *set notation* or *interval notation*. These notations are presented in the table below:

Interval	Set Notation	Graph
(a, b)	$\{x \mid a < x < b\}$	
$[a, b]$	$\{x \mid a \leq x \leq b\}$	
$[a, b)$	$\{x \mid a \leq x < b\}$	
$(a, b]$	$\{x \mid a < x \leq b\}$	
(a, ∞)	$\{x \mid a < x\}$	
$[a, \infty)$	$\{x \mid a \leq x\}$	
$(-\infty, b)$	$\{x \mid x < b\}$	
$(-\infty, b]$	$\{x \mid x \leq b\}$	
$(-\infty, \infty)$	$\mathbb{R}$ (set of all real numbers)	

Solutions to inequalities in this module will be represented using *set notation.*

Example 1: Solve the inequality $\dfrac{2x}{x+1} \geq 1$.

Solution:

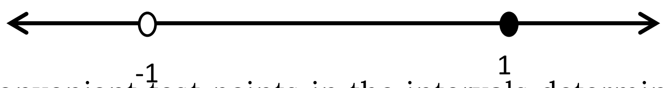
(a) Rewrite the inequality as a single rational expression.

$$\frac{2x}{x+1} - 1 \geq 0$$

$$\frac{2x-(x+1)}{x+1} \geq 0$$

$$\frac{x-1}{x+1} \geq 0$$

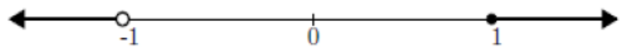
(b) The rational expression will be zero for x = 1 and undefined for x = -1. The value x = 1 is included while x = -1 is not. Mark these on the number line. Use a shaded circle for x = 1 (a solution and an unshaded circle for x = -1(not a solution).



(c) Choose the convenient test points in the intervals determined by -1 and 1 to determine the sign of $\frac{x-1}{x+1}$ in these intervals. Construct a table of signs as shown below.

Interval	$x < -1$	$-1 < x < 1$	$x > 1$
Test Point	$x = -2$	$x = 0$	$x = 2$
$x-1$	-	-	+
$x+1$	-	+	+
$\frac{x-1}{x+1}$	+	-	+

(d) Since we are looking for the intervals where the rational expression is positive or zero, we determine the solution to be the set $\{x \in R \mid x < -1 \; or \; x \geq 1\}$. Plot this set on the number line.



Example 2. Solve the inequality $\frac{3}{x-2} - \frac{1}{x} < 0$.

Solution:

(a) Rewrite the inequality with zero on one side.

$$\frac{3}{x-2} - \frac{1}{x} < 0$$

$$\frac{3x-(x-2)}{x(x-2)} < 0$$

$$\frac{2x+2}{x(x-2)} < 0$$

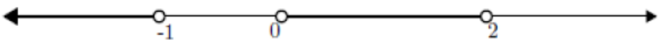
$$\frac{2(x+1)}{x(x-2)} < 0$$

(b) The rational expression will be zero for x = -1 and undefined for 0 and 2. Plot these points on a number line. Use hollow circles since these values are not part of the solution.

(c) Construct a table of signs to determine the sign of the function in each interval determined by -1, 0 and 2.

Interval	$x < -1$	$-1 < x < 1$	$0 < x < 2$	$x > 2$
Test Point	$x = -2$	$x = -\frac{1}{2}$	$x = 1$	$x = 3$
$2(x+1)$	-	+	+	+
x	-	-	+	+
$x-2$	-	-	-	+
$\frac{2(x+1)}{x(x-2)}$	**-**	**+**	**-**	**+**

(d) Summarize the intervals satisfying the inequality. The solution set of the inequality is the set $\{x \in R \mid x < -1 \text{ or } 0 < x < 2\}$. Plot this set on the number line.



1.4 Rational Function Represented as Table of Values, Graph and Equation

Rational Functions can be represented in three ways, namely: a table of values, an equation, and a graph.

Example Consider a 100-meter track used for foot races. Represent the speed of a runner as a function of the time it takes to run 100 meters in the track through (a) equation, (b) table of values and (c) graph.

a. Equation

Solution. Since the speed of a runner depends on the time it takes to run 100 meters, we can represent speed as a function of time.

Let x represent the time it takes to run 100 meters. Then the speed can be represented as a function $s(x)$ as follows:

$$s(x) = \frac{100}{x}$$

Observe that it is similar to the structure to the formula $s = \frac{d}{t}$ relating speed, distance, and time.

b. Table of Values

Solution. A table of values can help us determine the behavior of a function as the variable changes.

Let x be the runtime and $s(x)$ be the speed of the runner in meters per second, where $(x) = \frac{100}{x}$. The table of values for run times from 10 seconds to 20 seconds is as follows:

x	10	12	14	16	18	20
$s(x)$	10	8.33	7.14	6.25	5.56	5

From the table we can observe that the speed decreases with time.

c. Graph

Plot the points on the table of values on a Cartesian plane. Determine if the points on the function $s(x) = \frac{100}{x}$ follow a smooth curve or a straight line.

Solution. Assign points on the Cartesian plane for each entry on the table of values above:

A(10,10) B(12,8.33) C(14, 7.14) D(16, 6.25) E(18,5.56) F(20,5)

Plot these points on the Cartesian plane:

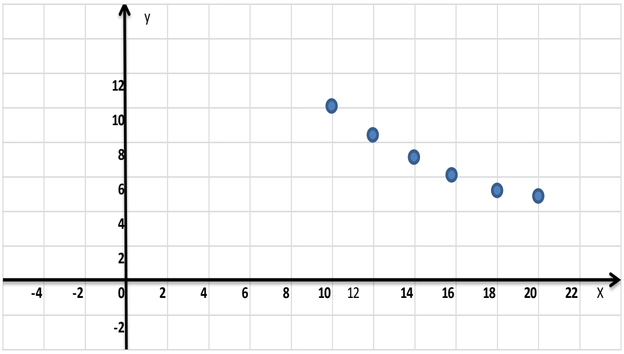


Figure 1

By connecting the points, we can see that they are not collinear but rather follows a smooth curve.

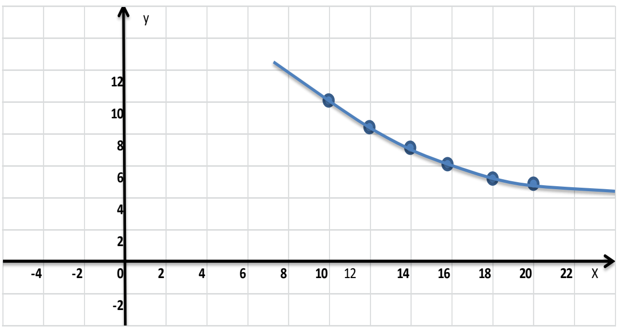


Figure 2

For the 100-meter dash scenario, we have constructed a function of speed against time, and represented our function with a table of values and a graph.

The previous example is based on a real world scenario and has limitations on the values of the x-variable. For example, a runner cannot have negative time (which would mean he is running backwards in time!), nor can he exceed the limits of human physiology (can a person run 100-meters in 5 seconds?). However, we can apply the skills of constructing tables of values and plotting graphs to observe the behaviour of rational functions.

Example2. Represent the rational function given by $f(x) = \frac{x-1}{x+1}$ using a table of values and plot a graph of the function by connecting points.

Solution. Since we are now considering functions in general, we can find function values across more values of *x*. Let us construct a table of values for some *x*-values from 10 to 10:

x	-10	-8	-6	-4	-2	0	2	4	6	8	10
$f(x)$	1.22	1.29	1.4	1.67	3	-1	0.33	0.6	0.71	0.78	0.82

Let us attempt to get a better picture by plotting the points on a Cartesian plane and connecting the points.

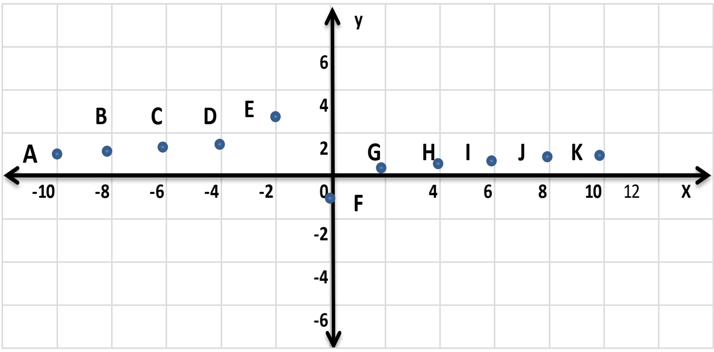


Figure 3

Observe that the function will be undefined at $x = -1$. This means that there cannot be a line connecting point E and point F as this implies that there is a point in the graph of the function where $x = -1$. This means that $x = -1$ is a **vertical asymptote**, where the graph of the function will never cross or touch but will tend to get closer and closer without bound.

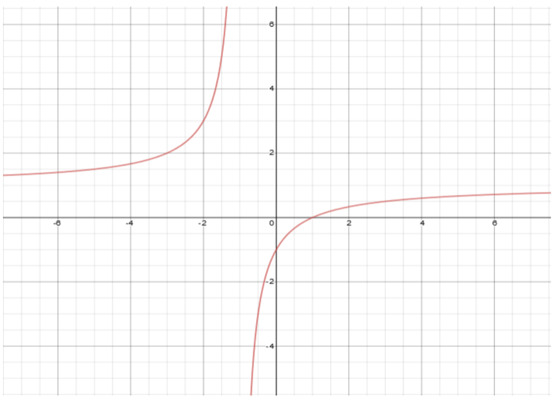


Figure 5:

1.5 Domain and Range of Rational Function

The **domain** of a rational function $f(x) = \frac{P(x)}{Q(x)}$ is all values of x that will not make the $Q(x)$ equal to zero. To find the domain of the Rational Function, set the denominator equal to zero and solve for x.

The **range** of a rational function are all the possible resulting values of the dependent variables after we have substituted the domain. To find the Range of the Rational Function:

(a) Find the Inverse of the Function
(b) Find the Domain of the Inverse Function
(c) State the Range.

Example 1: Find the domain and range of $f(x) = \frac{1}{x}$.

To find the Domain:

$x \neq 0.$ → *The Denominator cannot be equal to zero*

$D(f): \{x \in \mathbb{R} | x \neq 0\}$ → Read as *The Domain of function is x is an element of all real number such that x is not equal to zero.*

To find the Range:

$f(x) = \frac{1}{x}$ → Step 1: Find the inverse of the rational function.

$y = \frac{1}{x}$ → Change f(x) to y

$x = \frac{1}{y}$ → Interchange x and y

$(y)x = \frac{1}{y}(y)$ → Solve for y in term of x.

$xy = 1$

$\frac{xy}{x} = \frac{1}{x}$

$y = \frac{1}{x}$

$f^{-1} = \frac{1}{x}$ → This is the inverse function

$x \neq 0$ → Step 2: Find the Domain of the Inverse Function

$D(f^{-1}): \{x \in \mathbb{R} | x \neq 0\}$ → Domain of the inverse function

$R(f): \{y \in \mathbb{R} | y \neq 0\}$ → Step 3: State the Range of the function. *Note: The Domain of the Inverse Function is the Range of the Original Function.*

Example 2: Find the domain and range of $f(x) = \frac{x+2}{x-2}$.

To find the Domain:

$x - 2 \neq 0$ → The Denominator cannot be equal to zero

$x \neq 2$

$D(f): \{x \in \mathbb{R} | x \neq 2\}$ → Read as *The Domain of function is x is an element of all real number such that x is not equal to two.*

To find the Range:

$f(x) = \frac{x+2}{x-2}$ → Step 1: Find the inverse of the rational function.

$y = \frac{x+2}{x-2}$ → Change f(x) to y

$x = \frac{y+2}{y-2}$. → Interchange x and y

$(y-2)x = \left(\frac{y+2}{y-2}\right)(y-2)$ → Solve for y in term of x.

$xy - 2x = y + 2$

$xy - y = 2x + 2$

$y(x-1) = 2x + 2$

$\frac{y(x-1)}{(x-1)} = \frac{2x+2}{x-1}$

$y = \frac{2x+2}{x-1}$

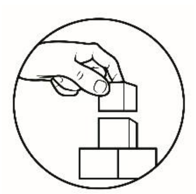
$f^{-1} = \frac{2x+2}{x-1}$ → This is the inverse function

$x - 1 \neq 0$ → Step 2: Find the Domain of the Inverse Function

$x \neq 1$

$D(f^{-1}): \{x \in \mathbb{R} | x \neq 1\}$ → Domain of the inverse function

$R(f): \{y \in \mathbb{R} | y \neq 1\}$ → Step 3: State the Range of the function.
Note: The Domain of the Inverse Function is the Range of the Original Function.



What's More

Activity 1 Rational Function Defined

Tell which of the following defines a rational function.

1. $f(x) = -3^x$
2. $f(x) = \left(\frac{2}{x-1}\right)^3$
3. $f(x) = \frac{x}{x-1}$

Activity 2 Real-Life Situations and Rational Functions

Tell which of the following situations illustrate a rational function.

1. A river flows at 2.8 kilometer per hour and Anita takes 8 hours to row 20 kilometer up the river and back. The relation between her speed in rowing as a function of time.
2. The amount of electric current flows as a function of the load's resistance.
3. The amount of charged fees as a function of the number of sent messages in a mobile phone.

Activity 3 What I Am!

Determine whether the given is a rational function, a rational equation, a rational inequality or none of these.

1. $\sqrt{x} = x + 1$
2. $13x - 5 < 10 - 2x$
3. $4 = \dfrac{2x}{x+5}$

Activity 4. What's My Solution Set?

1. Solve: $\dfrac{x+3}{x-4} = \dfrac{x-5}{x+4}$
2. Solve: $\dfrac{4}{2x-1} \geq \dfrac{1}{x+1}$

Activity 5. Complete Me!

Given the Rational functions, complete the table of values for $x = -2, -1, 0, 1, 2$

x	-2	-1	0	1	2
$f(x) = \dfrac{x}{x-1}$					
$f(x) = \dfrac{3}{x-3}$					
$f(x) = \dfrac{x+1}{x+2}$					

Activity 6 What's our Domain and Range?

Give the domain and range of the following rational function.

Rational Function	Domain	Range
$f(x) = \dfrac{2x+3}{4x-7}$		

What I Have Learned

Fill in the blanks.

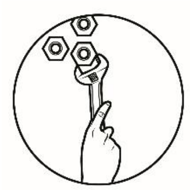
1. A rational function is a function of the form ____________________ where p(x) and q(x) are polynomial function and q(x) ≠ 0.

2. The domain of f(x) is the set of all value of real numbers except the value of the variable that makes the denominator equal to ____.

3. To simplify the equation $\frac{1}{x}+\frac{4}{x-1}=\frac{2}{3}$, we multiply both sides of the equation with LCD of ___________.

4. Like any other functions, rational functions can be represented through table of values, equation and _________.

5. The range of a rational function are all the possible resulting values of the the variable ______ the domain are substituted in the function.



What I Can Do

Use your learning on Rational Functions to answer the following:

1. In an organ pipe, the frequency $\boldsymbol{f}$ of vibration of air is inversely proportional to the length **L** of the pipe. Suppose that the frequency of vibration in a 10 foot pipe is 54 vibrations per second. Express f as a function of L.

2. For certain medicines, health care professionals may use Young's Rule to estimate the proper dosage for a child when the adult dosage. Young's Rule is given by $C=\frac{y}{y+12}\cdot A$ where C represents the child's dose, D represents the adult dose, and y represents the child age in years. Estimate the dosage of amoxicillin for an 8 years old child if the adult dosage is 250 mg.

Assessment

Multiple Choice: Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. Which of the following is NOT a rational function?

   A. $f(x) = \frac{1-x}{x}$ C. $f(x) = \frac{x}{2-x}$

   B. $f(x) = \frac{4}{x}$ D. $f(x) = \frac{3x}{2}$

2. A bus travels a distance of 250 meters. Express velocity **v** as a function of travel time **t**, in seconds.

   A. $v = \frac{250}{t}$ C. $v = 250t$

   B. $v = \frac{t}{250}$ D. $v = 250^t$

3. Which of the following relationships of physical quantities can be modeled by rational function?
   A. The area of a circle related to its radius.
   B. The electric current related to the resistance of a wire.
   C. The distance traveled related to speed of a car.
   D. The age of a man related to time spent in living.

Fot items 4 to 6, identify whether the given mathematical statement is a Rational Function, Rational Equation, Rational Inequality or None of these. Choices are provided nside the box. Write the chosen letter on a separate sheet of paper.

A. Rational Function	C. Rational Inequality
B. Rational Equation	D. None of these

4. $\frac{4}{5-x} = 7$ 5. $f(x) = \frac{2}{x}$ 6. $1 - x < \frac{1}{2x}$

7. Which of the following is NOT a solution to the inequality $\frac{2}{x+3} > \frac{x}{2}$?

   A. -3 C. -1
   B. -2 D. 0

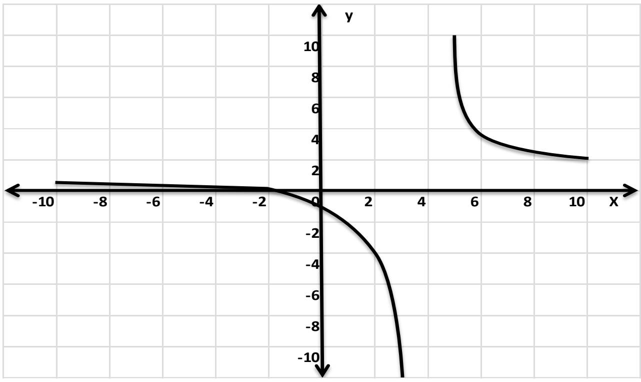
8. What is the solution set of the inequality $\frac{5x}{x-1} < 4$?

   A. $\{x|x \in R, -4 < x < 1\}$ C. $\{x|x \in R, x > 1\}$
   B. $\{x|x \in R, x < 1\}$ D. $\{x|x \in R, x \neq 1\}$

9. Which is a solution of the equation $x-\frac{18}{x}=3$?

A. 3
B. 4
C. 5
D. 6

10. Which equation best represents the graph?



A. $f(x)=\frac{x-2}{x-4}$
B. $f(x)=\frac{x+4}{x-2}$
C. $f(x)=\frac{x+2}{x+4}$
D. $f(x)=\frac{x+2}{x-4}$

11. Which rational function is represented by the table of values below?

x	1	2	3	-1	-2
$f(x)$	$\frac{4}{3}$	$\frac{8}{5}$	$\frac{12}{7}$	4	$\frac{8}{3}$

A. $f(x)=\frac{2x}{4+x}$
B. $f(x)=\frac{2x}{2+x}$
C. $f(x)=\frac{4x}{2x+1}$
D. $f(x)=\frac{2+x}{4+x}$

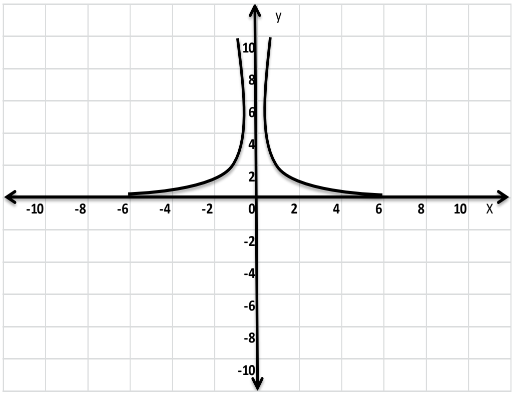
12. The domain of the function $f(x)=\frac{3-x}{x}$ is the set of all real numbers except __________.

A. -11
B. -1
C. 0
D. 11

13. What is the range of the functon in item 12?

A. all real numbers except -11
B. all real numbers except -1
C. all real numbers except 0
D. all real numbers except 11

14. Which family of functions does $f(x)=\frac{4}{x^2}$ belong to?



A. Rational

B. Trigonometric

C. Exponential

D. Logarithmic

15. What value of x in $f(x)=\dfrac{5-x}{x+3}$ will make the function undefined?

A. $-\dfrac{3}{5}$

B. -3

C. 3

D. 5

Additional Activity

Given the function $f(x)=\dfrac{x-3}{x+4}$ for $-6 \le x \le 2$,

a. Construct a table of values for the function for $-6 \le x \le 2$, x taking on integer values.

b. Identify values of x where the function will be undefined.

c. Plot the points corresponding to values in the table. Connect these points with a smooth curve. Explain why the points cannot be joined by a single curve. Identify the zeroes of this function.

What I Know

1. D
2. A
3. B
4. C
5. A
6. D
7. D
8. A
9. A
10. C
11. D
12. D
13. B
14. A
15. B

What's In

1. $\frac{1}{2}$
2. 2
3. 4
4. 1
5. $-\frac{2}{11}$

What's New

1. x = 10, 20, 100, 500, 1000
   y= 10000, 5000, 200, 100
2. $y = \frac{100,000}{x}$

What's More

A.1.

1. Not a Rational Function
2. Rational Function
3. Rational FunctionB

A.2.

1. Rational Function
2. Rational Function
3. Not a Rational Function

A.3.

1. None of these
2. Rational Inequality
3. Rational Equation

A.4

1. $x = \frac{1}{2}$
2. $-\frac{5}{2} \le x < -1$ or $x > \frac{1}{2}$

A.5

	-2	-1	0	1	2
$f(x)=\frac{x}{x-1}$	2/3	1/2	0	undefined	2
$f(x)=\frac{3}{x-3}$	-3/5	-3/4	-1	-3/2	-3
$f(x)=\frac{x+1}{x+2}$	undefined	0	1/2	2/3	3/4

A.6

Domain: $\left\{x \middle| x \neq \frac{7}{4}\right\}$

Range: $\left\{y \middle| y \neq \frac{1}{2}\right\}$

Answer Key

References

...u of Learning Resources (DepEd-BLR) (2016) ...*laterial.* Lexicon Press Inc., Philippines

...u of Learning Resources (DepEd-BLR) (2016) ...*uide.* Lexicon Press Inc., Philippines

What I Have Learned

1. $f(x)=\frac{p(x)}{q(x)}$
2. Zero (0)
3. $3x(x-1)$
4. Graph
5. y

What I Can Do

1. $f=\frac{540}{L}$
2. 100 mg

Assessment

16. D
17. A
18. B
19. B
20. A
21. C
22. A
23. A
24. D
25. D
26. C
27. C
28. B
29. A
30. B

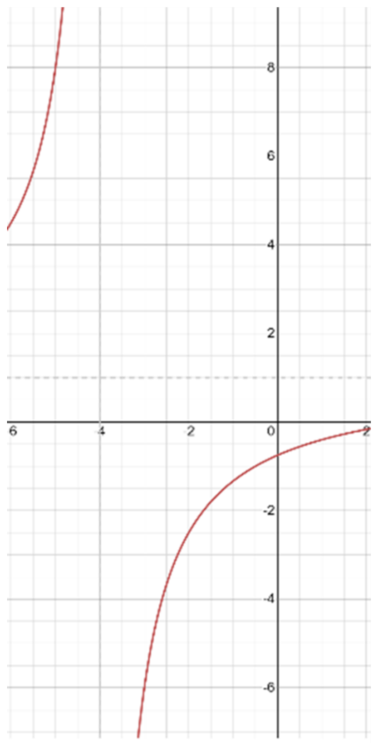
Additional Activities

a.

	-6	-5	-4	-3	-2	-1	0	1	2
$f(x)=\frac{x-3}{x+4}$	9/2	8	Undefined	-6	-3	-4/3	-3/4	-2/5	-1/6

b. x = 4

c.



References

1. Department of Education-Bureau of Learning Resources (DepEd-BLR) (2016) *General Mathematics Learner's Material.* Lexicon Press Inc., Philippines

2. Department of Education-Bureau of Learning Resources (DepEd-BLR) (2016) *General Mathematics Teacher's Guide*. Lexicon Press Inc., Philippines

EDITOR'S NOTE

This Self-learning Module (SLM) was developed by DepEd SOCCSKSARGEN with the primary objective of preparing for and addressing the new normal. Contents of this module were based on DepEd's Most Essential Learning Competencies (MELC). This is a supplementary material to be used by all learners of Region in all public schools beginning SY 2020-2021. The process of LR development was observed in the production of this module. This is Version 1.0. We highly encourage feedback, comment, and recommendations.

For inquiries or feedback, please write or call:

Department of Education – SOCCSKSARGEN
Learning Resource Management System (LRMS)

Regional Center, Brgy. Carpenter Hill, City of Koronadal

Telefax No.: (083) 2288825/ (083) 2281893

Email Address: region12@deped.gov.ph